Consequently, it is suggested that modelers use site-specific SOD rates. *In situ* methods such as described in Whittemore (1984a) and Markert et al. (1983) are more useful and credible than laboratory methods at this time.

As an aid to estimating SOD rates and establishing reasonable ranges for calibration, the SOD literature values in Tables 3-26, 3-27, and 3-28 are presented for rivers and streams, lakes and reservoirs, and estuaries and marine environments, respectively. These should be considered only as order of magnitude estimates.

3.6 PHOTOSYNTHESIS AND RESPIRATION

3.6.1 Introduction

Photosynthetic oxygen production ($P$) and respiration ($R$) can be important sources and sinks of dissolved oxygen in natural waters. Many models simulate these processes directly in terms of algal growth and respiration. For example, net algal growth is simulated with the QUAL-II model (Roesner et al., 1981) using:

$$\frac{\partial A}{\partial t} = (\mu - \rho - \sigma)A \quad (3-63)$$

where $A$ = algal concentration, mass/volume
$\mu$ = specific growth rate of algae, 1/time
$\rho$ = algal respiration rate, 1/time
$\sigma$ = algal settling rate, 1/time

The net algal oxygen production minus consumption is simulated by QUAL-II as:

$$\frac{\partial C}{\partial t} = (a_1\mu - a_2\rho)A \quad (3-64)$$

where $C$ = dissolved oxygen concentration, mass/volume
\[ \alpha_1 \] = oxygen production per unit of algal mass, mass oxygen/mass algae
\[ \alpha_2 \] = oxygen uptake per unit of algal mass, mass oxygen/mass algae

The stoichiometric coefficients \( \alpha_1 \) and \( \alpha_2 \) relate algal growth and death to oxygen production and consumption. Tables 3-29 and 3-30 summarize values of these coefficients used in different models.

**TABLE 3-26. MEASURED VALUES OF SEDIMENT OXYGEN DEMAND IN RIVERS AND STREAMS**

<table>
<thead>
<tr>
<th>SOD, ( \text{gO}_2/\text{m}^2 \text{ day} )</th>
<th>Environment</th>
<th>Experimental Conditions</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.022-0.92</td>
<td>Upper Wisconsin River</td>
<td>60-hour laboratory core incubation, periodic mixing, 4(^\circ)C, dark</td>
<td>Sullivan et al. (1978)</td>
</tr>
<tr>
<td>0.09±0.02 ( (81\text{C}) )</td>
<td>Eastern U.S. River</td>
<td>45 day incubation of 0.6 liters sediment in 3.85 liters BOD dilution water, tight</td>
<td>NCASI (1981)</td>
</tr>
<tr>
<td>0.15±0.04 ( (62\text{C}) )</td>
<td>Southeastern U.S. River</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.20±0.03 ( (62\text{C}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.29±0.07 ( (68\text{C}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.18±0.05 ( (62\text{C}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.55±0.22 ( (62\text{C}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.60±0.28 ( (62\text{C}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.87±0.23 ( (63\text{C}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.2-5.7</td>
<td>Fresh shredded tree bark</td>
<td>10-liter incubations in aged tap water, room temperature, light</td>
<td>NCASI (1971)</td>
</tr>
<tr>
<td>0.52-3.6</td>
<td>Aged shredded tree bark</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-33</td>
<td>Four eastern U.S. rivers downstream of paper mill discharges</td>
<td>In-situ chamber respirometers, 22-27(^{\circ})C, light, stirred at varying rates; open-ended tungel respirometer, in-situ, 22-27(^{\circ})C, dark</td>
<td>NCASI (1978)</td>
</tr>
<tr>
<td>0.9-14.1</td>
<td>Eastern U.S. river downstream of paper mill discharge</td>
<td>In-situ respirometer stirred at various rates 9-16(^{\circ})C, dark, ( \Theta = 1.08 )</td>
<td>NCASI (1979)</td>
</tr>
<tr>
<td>&lt;0.1-1.4 ( (62\text{C}) )</td>
<td>Northern Illinois rivers ( (N = 89 \text{ stations}) )</td>
<td>In-situ respirometry, dark, ( T = 59 - 31^{\circ})C time = 14 - 3 hours</td>
<td>Butts &amp; Evans (1978)</td>
</tr>
<tr>
<td>0.27-9.8</td>
<td>Six stations in eastern Michigan rivers</td>
<td>In-situ respirometry in stirred chambers, 15-27 hours dark, 19-25(^{\circ})C, ( \Theta = 1.08 )</td>
<td>Chiararo &amp; Burke (1980)</td>
</tr>
<tr>
<td>0.10±5.30 ( (62\text{C}) )</td>
<td>New Jersey rivers ( (10 \text{ stations}) )</td>
<td>In-situ respirometry, dark, 30 minutes-8 hours, stirred, temperature unknown</td>
<td>Hunter et al. (1973)</td>
</tr>
<tr>
<td>1.1-12.8</td>
<td>Swedish rivers</td>
<td>In-situ respirometry, light, stirred, 0-10(^{\circ})C</td>
<td>Edberg &amp; Hofsten (1973)</td>
</tr>
<tr>
<td>0.3-1.4</td>
<td>Swedish rivers</td>
<td>Laboratory incubations, stirred, dark, 5-10(^{\circ})C</td>
<td>Edberg &amp; Hofsten (1973)</td>
</tr>
<tr>
<td>0.20-1.2</td>
<td>Spring Creek, PA</td>
<td>Laboratory incubations in dark, stirred, 20(^{\circ})C</td>
<td>McDonnell &amp; Hall (1969)</td>
</tr>
<tr>
<td>1.5-9.8</td>
<td>74 samples from 21 English rivers</td>
<td>Laboratory incubation of cores; 15(^{\circ})C</td>
<td>Rolley &amp; Owens (1967)</td>
</tr>
<tr>
<td>4.6-44.</td>
<td>Streams</td>
<td>Oxygen mass balance</td>
<td>James (1974)</td>
</tr>
<tr>
<td>SOD, gO₂/m²/day</td>
<td>Environment</td>
<td>Experimental Conditions</td>
<td>References</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------------------------</td>
<td>----------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1-7</td>
<td>Green Bay, Lake Michigan</td>
<td>Lab incubation in darkness, 20°C</td>
<td>Gardiner et al. (1984)</td>
</tr>
<tr>
<td>0-2.2</td>
<td>Fish culture ponds</td>
<td><strong>In situ respirometer with 100-cm long plexiglass columns (dark pvc), over 47 days. Temperature unknown.</strong></td>
<td>Shapiro &amp; Zur (1981)</td>
</tr>
<tr>
<td>0.4-2.6</td>
<td>Swedish lakes</td>
<td><strong>In situ respirometer, light stirred, 5-18</strong></td>
<td>Edberg &amp; Hofsten (1973)</td>
</tr>
<tr>
<td>0.21-1.5</td>
<td>Swedish lakes</td>
<td>Laboratory incubations, stirred, dark, 10-13°C</td>
<td>Edberg &amp; Hofsten (1973)</td>
</tr>
<tr>
<td>5.5 (31-32.5°C)</td>
<td>Horseshoe Lake, IL</td>
<td><strong>In situ respirometer, dark. Stirred about 1 hour</strong></td>
<td>Butts &amp; Evans (1979)</td>
</tr>
<tr>
<td>5.1 (22.5-25.0°C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1 (13.2-16.0°C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.04-3.3</td>
<td>Lake Apopka, FL</td>
<td>Laboratory incubation of cores at room temperature, 2-3 hours, light. No stirring. Laboratory flow-through system (closed, 100 l volume)</td>
<td>Belanger (1981)</td>
</tr>
<tr>
<td>0.4-3.6</td>
<td>Lake Apopka, FL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40-0.45</td>
<td>Hyrum Reservoir, UT</td>
<td>3-phase microcosms, 25°C, dark</td>
<td>Medine et al. (1980)</td>
</tr>
<tr>
<td>0.27</td>
<td>Lake Powell</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.12-0.22</td>
<td>Shagawa Lake</td>
<td><strong>In-situ chambers (1 m²), at 7-12 m depths; 12-14°C (est.)</strong></td>
<td>Sonzogni et al. (1977)</td>
</tr>
<tr>
<td>0.47-0.92</td>
<td>Swedish Lakes</td>
<td>Laboratory measurement with undisturbed cores; used in situ temperatures</td>
<td>Granelli (1977)</td>
</tr>
<tr>
<td>0.72-8.40</td>
<td>Lakes</td>
<td>Oxygen mass balance</td>
<td>James (1974)</td>
</tr>
<tr>
<td>0.6-3.6</td>
<td>Hamilton Harbor, Lake Ontario</td>
<td><strong>In situ chambers, 11-16°C</strong></td>
<td>Polak &amp; Haffner (1978)</td>
</tr>
<tr>
<td>1.7-8.9</td>
<td>Lake Mohegan, NY</td>
<td>Measurement based on mass balance, continuous flow lab chamber, 22-32°C</td>
<td>Fillos (1977)</td>
</tr>
<tr>
<td>0.17-0.5</td>
<td>Swedish lakes</td>
<td><strong>In situ &amp; laboratory measurements, winter temperatures</strong></td>
<td>Edberg (1977)</td>
</tr>
<tr>
<td>0.54-0.71</td>
<td>Swedish lakes</td>
<td>Laboratory incubation of undisturbed cores, 8°C</td>
<td>Andersen (1977)</td>
</tr>
<tr>
<td>0.3-1.0</td>
<td>Lake Hartwell, SC</td>
<td>Laboratory chambers, 18°C</td>
<td>Brewer et al. (1977)</td>
</tr>
<tr>
<td>0.076-0.48</td>
<td>Marion Lake, BC</td>
<td>Laboratory incubation of undisturbed cores, no mixing, 15°C</td>
<td>Hargrave (1969)</td>
</tr>
<tr>
<td>0.004-0.012</td>
<td>Lake Superior</td>
<td>Laboratory incubation of undisturbed cores, 4°C</td>
<td>Glass &amp; Podolski (1975)</td>
</tr>
</tbody>
</table>

In addition to algal respiration, respiration from zooplankton and nekton can contribute to oxygen depletion, and would be included in Equation (3-64), along with additional equations to describe their growth and death. Models that simulate algae and zooplankton (such as those in Tables 3-29 and 3-30) are discussed in detail in Chapters 6 and 7 of this report. This section describes methods to predict P-R without simulating algal growth or respiration. The methods pertain largely to streams and rivers, and are useful in that they simplify the modeling approach.

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It should be mentioned that some water quality models do not simulate photosynthesis and algal respiration. This approach is valid where \( P=0 \) and \( R=0 \). Other models simulate only daily average photosynthetic oxygen production (\( \overline{P} \)) and daily average respiration (\( \overline{R} \)). If, on a daily average basis, \( \overline{P} - \overline{R} = 0 \), these models would predict little effect of algal activity on dissolved oxygen. However, if \( \overline{P} \) and \( \overline{R} \) are both large numbers, then actual dissolved oxygen levels will be higher during the day and lower at night than predicted by the models.

### 3.6.2 Methods

Table 3-31 summarizes the methods reviewed to predict photosynthetic oxygen production and respiration without simulating algal growth. The methods consist of either single station methods or two-station methods. Odum (1956) appears to be one of the first researchers to use this approach.

<table>
<thead>
<tr>
<th>SDP, ( gO_2/m^2) day</th>
<th>Environment</th>
<th>Experimental Conditions</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10\pm0.03 (( 12^\circ)C)</td>
<td>A North Carolina estuary</td>
<td>45 day incubation of 0.6 liters sediment in 3.85 liters BOD dilution water, light</td>
<td>NCASI (1981)</td>
</tr>
<tr>
<td>0.20\pm0.05 (( 20^\circ)C)</td>
<td>Buzzards Bay near raw sewage outfall</td>
<td>In-situ dark respirometers, stirred, 1-3 days. Temperature unknown</td>
<td>Smith et al. (1973)</td>
</tr>
<tr>
<td>0.22\pm0.09 (( 20^\circ)C)</td>
<td>Buzzards Bay control</td>
<td>In-situ dark respirometers, stirred, 1-3 days. Temperature unknown</td>
<td>Smith et al. (1973)</td>
</tr>
<tr>
<td>0.37\pm0.15 (( 36^\circ)C)</td>
<td>Puget Sound sediment cores</td>
<td>Laboratory incubations</td>
<td>Pamatmat et al. (1973)</td>
</tr>
<tr>
<td>2.32\pm0.16</td>
<td>Laboratory incubations</td>
<td>Pamatmat et al. (1973)</td>
<td></td>
</tr>
<tr>
<td>1.88\pm0.018</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.14-0.68 (( 5^\circ)C)</td>
<td>San Diego Trough (deep marine sediments)</td>
<td>In-situ respirometry for 5-13 hours, 4(^\circ)C, light</td>
<td>Smith (1974)</td>
</tr>
<tr>
<td>0.20-0.76 (( 10^\circ)C)</td>
<td>Yaquina River estuary, Oregon</td>
<td>Dark laboratory incubators, stirred, 20(^\circ)C</td>
<td>Martin &amp; Bella (1971)</td>
</tr>
<tr>
<td>0.30-1.52 (( 15^\circ)C)</td>
<td>Eastern tropical Pacific</td>
<td>Shipboard incubations, 15(^\circ)C stirred, dark</td>
<td>Pamatmat (1971)</td>
</tr>
<tr>
<td>0.05-0.10</td>
<td>The Baltic Sea</td>
<td>In-situ light respirometer, stirred, 10(^\circ)C</td>
<td>Edberg &amp; Hofsten (1973)</td>
</tr>
<tr>
<td>1.25-3.9</td>
<td>The Baltic Sea</td>
<td>Laboratory incubations, stirred, dark, 10(^\circ)C</td>
<td>Edberg &amp; Hofsten (1973)</td>
</tr>
<tr>
<td>0.9-3.0</td>
<td>Delaware Estuary (22 stations)</td>
<td>In-situ dark respirometry, 13-14(^\circ)C</td>
<td>Albert (1983)</td>
</tr>
<tr>
<td>0.4-0.71</td>
<td>Fresh &amp; brackish waters, Sweden</td>
<td>In-situ respirometry, 0-18(^\circ)C Laboratory cores, 5-13(^\circ)C</td>
<td>Edberg &amp; Hofsten (1973)</td>
</tr>
</tbody>
</table>
### TABLE 3-29. OXYGEN PRODUCED PER MASS OF ALGAE

<table>
<thead>
<tr>
<th>Model</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOSAG3</td>
<td>1.4 - 1.8 ( \frac{mg \text{ O}_2}{mg \text{ algae (D.W.)}} )</td>
<td>Duke &amp; Masch (1973)</td>
</tr>
<tr>
<td>QUAL-II</td>
<td>1.4 - 1.8 ( \frac{mg \text{ O}_2}{mg \text{ algae (D.W.)}} )</td>
<td>Roesner et al. (1977)</td>
</tr>
<tr>
<td>WASP</td>
<td>2.67 mg \text{ O}_2/mg C</td>
<td>Di Toro &amp; Connolly (1980)</td>
</tr>
<tr>
<td>WASP</td>
<td>2.66 mg \text{ O}_2/mg C</td>
<td>O'Connor et al. (1981)</td>
</tr>
<tr>
<td></td>
<td>.133 mg \text{ O}_2/mg Chl-a</td>
<td>O'Connor et al. (1981)</td>
</tr>
<tr>
<td>WASP</td>
<td>2.67 mg \text{ O}_2/mg C</td>
<td>Thomann &amp; Fitzpatrick (1982)</td>
</tr>
<tr>
<td>LAKE ECO</td>
<td>1.6 mg \text{ O}_2/mg algae (D.W.)</td>
<td>Chen &amp; Orlob (1975)</td>
</tr>
<tr>
<td>WQRRS</td>
<td>1.6 mg \text{ O}_2/mg algae (D.W.)</td>
<td>Smith (1978)</td>
</tr>
<tr>
<td>AQUA-IV</td>
<td>1.6 - 2.66 mg \text{ O}_2/mg C</td>
<td>Baca &amp; Arnett (1976)</td>
</tr>
<tr>
<td>ESTECO</td>
<td>1.6 - 1.8 mg \text{ O}_2/mg algae (D.W.)</td>
<td>Brandes (1976)</td>
</tr>
<tr>
<td>EAM</td>
<td>1.24 mg \text{ O}_2/mg algae (D.W.)</td>
<td>Porcella et al. (1983)</td>
</tr>
<tr>
<td>EAM</td>
<td>1.6 mg \text{ O}_2/mg algae (D.W.)</td>
<td>Bowie et al. (1980)</td>
</tr>
<tr>
<td>EAM</td>
<td>1.24 mg \text{ O}_2/mg algae (D.W.)</td>
<td>Tetra Tech (1980)</td>
</tr>
<tr>
<td>DEM</td>
<td>1.6 mg \text{ O}_2/mg algae (D.W.)</td>
<td>Feigner &amp; Harris (1970)</td>
</tr>
<tr>
<td>Vermont-QUAL-II</td>
<td>1.4 - 1.8 mg \text{ O}_2/mg algae (D.W.)</td>
<td>JRB (1983)</td>
</tr>
</tbody>
</table>

**Note:**

D.W. = dry weight

Both numerical and analytical techniques have since been developed. The light-dark bottle technique and benthic chamber method are also included in the table.

As shown in Table 3-31, O'Connell and Thomas (1965) applied a total derivative approach for P-R calculation, and compared the results against a
second procedure using a submerged algal chamber. Respiration was corrected for oxygen consumption by bacterial oxidation. Figure 3-17 compares the two methods for a station on the Truckee River, and shows good agreement.

O'Connor and Di Toro (1970) use a half cycle sine wave or a Fourier series to find the time varying photosynthetic oxygen production rate. In

**TABLE 3-30. OXYGEN CONSUMED PER MASS OF ALGAE**

<table>
<thead>
<tr>
<th>Model</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOSAG 3</td>
<td>1.6 - 2.3 mg O$_2$/mg algae (D.W.)</td>
<td>Duke &amp; Masch (1973)</td>
</tr>
<tr>
<td>QUAL-II</td>
<td>1.6 - 2.3 mg O$_2$/mg algae (D.W.)</td>
<td>Roesner et al. (1977)</td>
</tr>
<tr>
<td>WASP</td>
<td>1.87 mg O$_2$/mg C$^1$</td>
<td>Di Toro &amp; Connolly (1980)</td>
</tr>
<tr>
<td>WASP</td>
<td>2.0 mg O$_2$/mg C</td>
<td>Thomann &amp; Fitzpatrick (1982)</td>
</tr>
<tr>
<td>WASP</td>
<td>2.0 mg O$_2$/mg C</td>
<td>O'Connor et al. (1981)</td>
</tr>
<tr>
<td></td>
<td>.10 mg O$_2$/mg Chl-a</td>
<td>O'Connor et al. (1981)</td>
</tr>
<tr>
<td>LAKE ECO</td>
<td>1.6 mg O$_2$/mg algae (D.W.)</td>
<td>Chen &amp; Orlob (1975)</td>
</tr>
<tr>
<td>WQRRS</td>
<td>1.6 - 2.0 mg O$_2$/mg algae (D.W.)</td>
<td>Smith (1978)</td>
</tr>
<tr>
<td>AQUA-IV</td>
<td>1.6 - 2.66 mg O$_2$/mg C</td>
<td>Baca &amp; Arnett (1976)</td>
</tr>
<tr>
<td>ESTECO</td>
<td>1.6 - 1.8 mg O$_2$/mg algae (D.W.)</td>
<td>Brandes (1976)</td>
</tr>
<tr>
<td>EAM</td>
<td>.95 mg O$_2$/mg algae (D.W.)</td>
<td>Porcella et al. (1983)</td>
</tr>
<tr>
<td>EAM</td>
<td>1.6 mg O$_2$/mg algae (D.W.)</td>
<td>Bowie et al. (1980)</td>
</tr>
<tr>
<td>EAM</td>
<td>.95 mg O$_2$/mg algae (D.W.)</td>
<td>Tetra Tech (1980)</td>
</tr>
<tr>
<td>DEM</td>
<td>1.6 mg O$_2$/mg algae (D.W.)</td>
<td>Feigner &amp; Harris (1970)</td>
</tr>
<tr>
<td>Vermont -</td>
<td></td>
<td></td>
</tr>
<tr>
<td>QUAL-II</td>
<td>1.6 - 2.3 mg O$_2$/mg algae (D.W.)</td>
<td>JRB (1983)</td>
</tr>
</tbody>
</table>

$^1$ This is multiplied by an oxygen limitation factor, $\frac{O_2}{K + O_2}$, where $K$ is a half-saturation constant equal to 0.1 mg/l.

Note:
D.W. = dry weight
<table>
<thead>
<tr>
<th>Source</th>
<th>Equations</th>
<th>Symbols</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odum (1956)</td>
<td>see comments</td>
<td>see comments</td>
<td>1. Photosynthetic oxygen production was based on a graphical procedure. Either two stations or single station approaches could be used. A method was also presented to find the respiration coefficient.</td>
</tr>
<tr>
<td>O'Connell and Thomas (1965)</td>
<td>( P - R = \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} - k_2(C - C_s) + k_1L + k_nN )</td>
<td>( U ) = stream velocity, ( k_2 ) = respiration rate, ( C ) = dissolved oxygen, ( C_s ) = dissolved oxygen saturation, ( k_1 ) = CBOD decay rate, ( L ) = CBOD, ( k_n ) = nitrification rate, ( N ) = NBOD</td>
<td>1. ( P-R ) was found in two independent ways. In the first, all terms in the dissolved oxygen mass-balance were found independently and then ( P-R ) was found as the only remaining term in the oxygen balance. In the second method, an algal chamber was placed on the river bed. 2. The two methods gave comparable results. 3. The approach was used on the Truckee River, where attached algae were abundant.</td>
</tr>
<tr>
<td>O'Connor and Di Toro (1970)</td>
<td>Half cycle sine wave: ( P = \frac{P}{2} \sin \left{ \frac{\pi (t-t_s)}{t_s+1} \right} ) ( t_s ) ( P_{(x)} = ) maximum rate of photosynthetic oxygen production, ( mg/(l-day) )</td>
<td>( P ) = rate of photosynthetic oxygen production, ( mg/(l-day) )</td>
<td>1. This approach is found in DIURNAL, a stream model developed by O'Connor and Di Toro. 2. The approach is potentially applicable to any vertically mixed water body. 3. The method of Erdmann (1979a) was used to evaluate ( P ) and ( R ) for a wasteload allocation application on the Shenandoah River (Deb and Bowers, 1983) and on Leatherwood Creek, Arkansas (Deb et al., 1989). 4. O'Connor and Di Toro (1970) applied the method to the Grand, Clinton, and Flint rivers in Michigan, the Truckee River in Nevada, and the Ivel River in Great Britain. They used a trial and error procedure to determine ( P_m, t_s, P ) and ( R ) to best fit observed diurnally varying dissolved oxygen data.</td>
</tr>
</tbody>
</table>

(continued)
<table>
<thead>
<tr>
<th>Source</th>
<th>Equations</th>
<th>Symbols</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kelly, Hornberger, Cosby (1975)</td>
<td>$P - R = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\omega t)$</td>
<td>$A_n =$ unknown coefficients $\omega = 2 \pi/48$</td>
<td>1. A 48-hour cycle was used so that values at the beginning and end of a day are not constrained to be identical.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The $A_n$ are determined based on measurements of dissolved oxygen at either one of two locations in a stream. They are chosen to give a &quot;best fit&quot; between predicted and observed dissolved oxygen values.</td>
<td>2. $R$ is total respiration, including both algal respiration and bacterial decay.</td>
</tr>
<tr>
<td>Hornberger and Kelly (1972)</td>
<td>$P - R = \frac{dC}{dt} + U \frac{dC}{dx} - k_2 (C_s - C)$</td>
<td>$C =$ dissolved oxygen concentration $U =$ stream velocity $k_2 =$ reaeration rate $C_s =$ dissolved oxygen saturation</td>
<td>3. The single station analysis can be used when the dissolved oxygen concentrations at the upstream and downstream stations are approximately the same.</td>
</tr>
<tr>
<td>Erdmann (1979a)</td>
<td>$P - R = k_2 (C_s - C) - \frac{dC}{dt}$</td>
<td>$C_{om} =$ concentration of dissolved oxygen at station $m$ and time $n$</td>
<td>1. Three methods were examined to predict $P-R$: a finite difference method, an analytical solution assuming $P-R$ remains constant over the time interval, and a second analytical method assuming $P-R$ varies linearly over a time step.</td>
</tr>
<tr>
<td></td>
<td>where: $\frac{dC}{dt} = 1/2 \left{ \frac{C_{11} - C_{21}}{t_2 - t_1} + \frac{C_{12} - C_{22}}{t_f - t_1} \right}$</td>
<td>$t_2 =$ time of sample at downstream station $t_1 =$ time of sample at upstream station</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t_f =$ time of sample at downstream station $k_2 =$ reaeration rate $C_{11} =$ dissolved oxygen saturation $C_{22} =$ dissolved oxygen saturation $C =$ dissolved oxygen</td>
<td>2. The analytical methods were preferred over the numerical approach from a conceptual point of view, and because time steps smaller than the residence time through the stream reach could be used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3. $R$ is total oxygen consumption rate by both algae and bacteria.</td>
</tr>
</tbody>
</table>
### TABLE 3-31. (continued)

<table>
<thead>
<tr>
<th>Source</th>
<th>Equations</th>
<th>Symbols</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erdmann (1979b)</td>
<td>[ P = (\Delta C_u + \Delta C_d) ]</td>
<td>( P ) = daily average photosynthesis</td>
<td>1. The method is a simplification of Erdmann (1979a) and is used to predict daily average values of ( P-R ) from data at two stations.</td>
</tr>
<tr>
<td></td>
<td>[ R = k_2 \left( -\frac{D_u + D_d}{2} \right) + (\Delta C_u + \Delta C_d) - \left( \frac{C_d - C_0}{t_f} \right) ]</td>
<td>( R ) = daily average respiration</td>
<td>2. The method was applied to the Charles River, Massachusetts.</td>
</tr>
<tr>
<td></td>
<td>( \Delta C_u, \Delta C_d ) = diurnal range of dissolved oxygen at upstream stations</td>
<td>( \Delta C_u, \Delta C_d ) = diurnal range of dissolved oxygen at upstream stations</td>
<td>3. Some important assumptions include constant temperature and symmetrical diurnal curves.</td>
</tr>
<tr>
<td></td>
<td>( D_u, D_d ) = daily average dissolved oxygen deficit at upstream and downstream stations</td>
<td>( D_u, D_d ) = daily average dissolved oxygen deficit at upstream and downstream stations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( C_u, C_d ) = daily average dissolved oxygen concentration at upstream and downstream stations</td>
<td>( C_u, C_d ) = daily average dissolved oxygen concentration at upstream and downstream stations</td>
<td></td>
</tr>
<tr>
<td>Gulliver, Mattke, Stefan (1982)</td>
<td>[ P - R = \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} - \frac{\partial}{\partial x} \left( \frac{\partial C}{\partial x} \right) - k_2 (C - C_s) ]</td>
<td>( U ) = stream velocity</td>
<td>1. A finite difference computer model DORM was used to route dissolved oxygen changes between two stations and includes the effects of temperature variations and dissolved oxygen levels on respiration.</td>
</tr>
<tr>
<td></td>
<td>( k_2 ) = reseration rate</td>
<td></td>
<td>2. The model was applied to experimental stream reaches in the U.S. EPA's Monticello Ecological Research Station, Minnesota.</td>
</tr>
<tr>
<td></td>
<td>( C ) = dissolved oxygen</td>
<td></td>
<td>3. For the channels analyzed, it was found that affects of longitudinal dispersion were negligible. However the results were sensitive to reseration, residence time between the two stations, and temperature dependent processes (saturation and respiration rates).</td>
</tr>
<tr>
<td></td>
<td>( C_s ) = dissolved oxygen saturation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{\partial}{\partial x} ) = longitudinal dispersion coefficient</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued)
<table>
<thead>
<tr>
<th>Source</th>
<th>Equations</th>
<th>Symbols</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. EPA (1983)</td>
<td>light and dark bottle technique</td>
<td></td>
<td>1. Light and dark bottles are suspended at various depths in water and dissolved oxygen measurements are made at regular intervals to determine P-R.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2. This method suffers from numerous limitations which include:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Only photosynthetic activity of algae in water column is measured</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- The estimate of R includes algal and bacterial respiration</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- The P-R is a point estimate, rather than representative of a reach.</td>
</tr>
<tr>
<td>U.S. EPA (1983)</td>
<td>benthic chamber</td>
<td></td>
<td>1. P-R of attached algae is measured using a clear benthic chamber and a covered (dark) chamber.</td>
</tr>
</tbody>
</table>
their applications, they used a trial and error procedure to determine P-R that best fit diurnally varying dissolved oxygen data. In the Deb and Bowers (1983) application of the same method, Erdmann's approach (1979a) was used to evaluate P-R. The method of Erdmann combines all terms which contribute to deoxygenation (algal respiration, CBOD decay and NBOD decay) into a single respiration term. To find algal respiration, CBOD and NBOD are subtracted from total community respiration.

Kelly et al., (1975), also shown in Table 3-31, use a Fourier series, but with a 48 hour period. The coefficients $A_n$ are not true Fourier coefficients but are based on a best fit between predicted and observed dissolved oxygen values. Cohen and Church (1981) have more recently applied these methods to measure productivity of algae in cultures open to the atmosphere.

Figure 3-17. Diurnal variation of (P-R) in Truckee River near Station 2B (O'Connell and Thomas, 1965).
Erdmann (1979a, 1979b) has developed methods to predict time-varying P-R values and daily average values. In the time varying case the concept of the Stokes total time derivative is used (see Figure 3-18). The total derivative is the sum of the time derivative ($\partial C/\partial t$) and the advective derivative ($U\partial C/\partial x$). The time derivative is evaluated as the average of two times, and the advective derivative is the average between two stations.

$$\frac{DC}{Dt} = \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x}$$

Figure 3-18. Concept of Stokes total time derivative. Here $DC/Dt = 0.43 \text{ mg } O_2/1\cdot \text{h}$ (from Erdmann, 1979a).

Gulliver et al., (1982) provide a literature review of the various methods used to predict P-R in streams. They also developed a computerized model to determine P-R that includes dispersion. However, they found that effects of dispersion were negligible for their applications. Several applications of diurnal curve analyses not reported in Table 3-31 include the work of Schurr and Ruchts (1977) who used a single station method to predict monthly average P-R values, and the work of Simonsen and Harremoes (1978) who used a two station approach to predict P-R on a river in Denmark.
The final two methods shown in Table 3-31 are the light-dark bottle method and the benthic chamber method. These methods measure P-R of algae in the water column (light-dark bottles) and of attached algae (benthic chamber). The methods provide single point estimates that may not be representative of the water body as a whole.

Some models simulate daily average photosynthetic oxygen production rather than time-varying production. Erdmann (1979b) shows that, the daily average photosynthesis oxygen products rates, \( P \), can be approximated by:

\[
\bar{P} = \frac{2 \Delta DO}{24} \text{ (mg/l/hr)}
\]  \hspace{1cm} (3-65)

where \( \Delta DO = \text{daily maximum dissolved oxygen concentration minus daily minimum dissolved oxygen concentration, mg/l} \)

This approximation appears to be valid only for reaeration rates less than 0.2/day (Manhattan College, 1983).

A second method of estimating \( P \) is to integrate a sinusoidal curve that represents the instantaneous photosynthetic oxygen production rate. The result is:

\[
\bar{P} = \frac{2 P_m p}{\pi} 
\]  \hspace{1cm} (3-66)

where \( P_m = \text{maximum daily photosynthetic oxygen production rate, mg/l/day} \)

\( p = \text{fraction of day when algae are producing oxygen, decimal fraction} \)

The U.S. EPA (1983) describes a third method to estimate daily average production based on light-dark bottle measurements:

\[
P = \frac{2P' \Delta T}{\cos(\pi t_1/f) - \cos(\pi t_2/f)}
\]  \hspace{1cm} (3-67)

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where $P' = \text{observed average production rate between times } t_2 \text{ and } t_1$

$\Delta T = (t_2 - t_1)/24$

$f = \text{number of hours in day when oxygen is being produced}$

Relationships between photosynthetic oxygen production and chlorophyll-a have been developed by a number of researchers. While a detailed review of these methods is outside of the scope of this section, several of the more commonly used formulations are summarized here. Megard et al. (1979) developed the following expression for daily average photosynthetic oxygen production:

$$\bar{P} = \frac{\ln\left(\frac{I_o}{I_z}\right) C_a P_m}{\varepsilon_C C_a + \varepsilon_W}$$  \hspace{1cm} (3-68)

where $I_o = \text{light intensity at the water surface}$

$I_z = \text{light intensity at depth } z$

$C_a = \text{chlorophyll-a concentration}$

$\varepsilon_C = \text{specific attenuation of light by chlorophyll-a}$

$\varepsilon_W = \text{specific attenuation of light by all causes other than chlorophyll-a}$

$P_m = \text{maximum daily photosynthetic oxygen production rate, mg/l/day}$

Demetracopoulos and Stefan (1983) modified this expression to predict hourly photosynthetic oxygen production, and used the expression in a model of the Mississippi River.

In experiments on the Sacramento-San Joaquin Estuary, Bailey (1970) correlated the daily photosynthetic oxygen production rate to a number of factors. The resulting expression was:

$$P_{av} = 3.16 C_a \left(\frac{I}{k_e}\right)^{0.677} + 0.16T - 0.56H$$  \hspace{1cm} (3-69)
where \( P_{av} \) = average daily gross photosynthetic rate, mg/l-day
\( I \) = mean daily solar intensity, cal/sq.cm-day
\( k_e \) = light extinction coefficient, 1/meter
\( T \) = mean temperature, \(^{0}C\)
\( H \) = mean water depth, m
\( C_a \) = mean chlorophyll, mg/l

Finally, simple relationships between chlorophyll-a and, \( P_m \) have been proposed (U.S. EPA, 1983). Figure 3-19 shows how \( P_m/Ca \) ratios are influenced by water temperature and algal carbon/Ca ratios. For a typical water temperature (20\(^{0}C\)) and a typical carbon/Ca ratio (50), \( P_m/Ca = 0.25 \). However, this ratio is likely to vary between 0.1 to 0.6 for the range of conditions present in streams.
3.6.3 Data

Table 3-32 summarizes data reviewed on photosynthetic oxygen production and respiration. Respiration is sometimes reported as total community respiration and at other times as algal respiration. As shown by the data, photosynthetic oxygen production can be quite variable, both over distance and time. In the Havelse River, for example, average photosynthetic oxygen production rates varied from 0.2 to 25.9 g/(m²-day). One of the primary reasons for the variability was because solar radiation intensity changed by more than an order of magnitude between measurement periods.

3.6.4 Summary

Most water quality models that simulate photosynthetic oxygen production and algal respiration simulate algal growth and respiration. Stoichiometric coefficients are used to convert growth and respiration to oxygen production and consumption. Tables 3-29 and 3-30 summarize these coefficients.

Some river water quality models use the approach that photosynthetic oxygen production and respiration can be modeled without the necessity of simulating algal activity. Rather, some type of curve, such as a sine curve or more generally a Fourier series, is used instead, where certain parameters must be delineated to characterize the curve.

Typically instream dissolved oxygen measurements at two stations are used to generate P-R data. Either finite difference or continuous solutions to dissolved oxygen mass balance equations are used. While light-dark bottles or benthic chambers can in principal be used to find the required information, these approaches are limited in a number of ways. The two station methods are better in that they provide an integrated estimate of algal activity.

However, two station methods should also be used cautiously. In a sense, the methods are curve fitting techniques: they are used to fit a
curve based on dissolved oxygen variation between two stations. Typically other rate constants such as reaeration rates, carbonaceous BOD decay, nitrogenous BOD decay are needed to fit the curves. Thus errors in these coefficients are propagated into P-R calculations. Also care should be taken if results are extrapolated to other situations (e.g., different temperatures, different solar intensities, and different nutrient loadings).

TABLE 3-32. PHOTOSYNTHETIC OXYGEN PRODUCTION AND RESPIRATION RATES IN RIVERS

<table>
<thead>
<tr>
<th>Reference</th>
<th>River</th>
<th>T (°C)</th>
<th>Pn (g/m²-day)</th>
<th>Pav (g/m²-day)</th>
<th>R (g/m²-day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O'Connor and Di Toro (1970)</td>
<td>Clinton, Michigan</td>
<td>21</td>
<td>13.2 - 22.9</td>
<td>4.2 - 7.3</td>
<td>9.3</td>
</tr>
<tr>
<td>O'Connor and Di Toro (1970)</td>
<td>Truckee, Nevada</td>
<td>28</td>
<td>12.9 - 26.</td>
<td>4.8 - 9.6</td>
<td>3.6 - 6.2</td>
</tr>
<tr>
<td>O'Connor and Di Toro (1970)</td>
<td>Ivel, Great Britain</td>
<td>16</td>
<td>24.</td>
<td>9.0</td>
<td>4.6</td>
</tr>
<tr>
<td>Thomas and O'Connell (1977)</td>
<td>Laboratory Streams</td>
<td>-</td>
<td>-</td>
<td>3.4 - 4.0</td>
<td>2.4 - 2.9</td>
</tr>
<tr>
<td>Erdmann (1979a,b)</td>
<td>Charles, Massachusetts</td>
<td>19-25</td>
<td>-</td>
<td>0.0 - 12.</td>
<td>0.0 - 36.</td>
</tr>
<tr>
<td>Deb and Bowers (1983)</td>
<td>Shenandoah, Virginia</td>
<td>23</td>
<td>4.8 - 17.4</td>
<td>-</td>
<td>0.9 - 5.9</td>
</tr>
<tr>
<td>Kelly et al. (1975)</td>
<td>Baker, Virginia</td>
<td>-</td>
<td>-</td>
<td>0.45</td>
<td>1.9</td>
</tr>
<tr>
<td>Kelly et al. (1975)</td>
<td>Rappahannock, Virginia</td>
<td>-</td>
<td>-</td>
<td>6.1</td>
<td>7.3</td>
</tr>
<tr>
<td>Kelly et al. (1975)</td>
<td>S. Fork Rivanna, Virginia</td>
<td>-</td>
<td>-</td>
<td>2.1</td>
<td>3.4</td>
</tr>
<tr>
<td>Kelly et al. (1975)</td>
<td>Rivanna, Virginia</td>
<td>-</td>
<td>-</td>
<td>2.3</td>
<td>5.4</td>
</tr>
<tr>
<td>Kelly et al. (1975)</td>
<td>South, Virginia</td>
<td>-</td>
<td>-</td>
<td>2.0</td>
<td>5.3</td>
</tr>
<tr>
<td>Kelly et al. (1975)</td>
<td>Mechums, Virginia</td>
<td>-</td>
<td>-</td>
<td>1.3</td>
<td>2.6</td>
</tr>
<tr>
<td>Simonsen and Harremoes (1978)</td>
<td>Havelse, Denmark</td>
<td>-</td>
<td>-</td>
<td>0.2 - 25.9</td>
<td>4.8 - 22.9</td>
</tr>
<tr>
<td>Gulliver et al. (1982)</td>
<td>Experimental Channels</td>
<td>9-24</td>
<td>5. - 45.</td>
<td>1.5 - 14.8</td>
<td>2.6 - 10.7</td>
</tr>
</tbody>
</table>

aAlgal respiration only
bTotal community respiration

Measurements were made over the period of one year, and solar radiation varied by more than a factor of 10.
In cases where diurnal water temperature changes are great, diurnal curve analyses should include temperature correction effects.

All of the approaches reviewed in Table 3-31 have apparently been successfully applied. However, no comprehensive comparison of the approaches against the same data set were found. In cases where a significant amount of data is available for analysis, a computerized approach such as Kelly et al. (1975) or Gulliver et al. (1982) appears to be better than trial and error procedures. The method that has been most rigorously tested is the DORM model of Gulliver et al. (1982). Also these methods can be used when the distance between stream stations is great, because the models do not assume that P-R remains constant over the travel time between the stations.

Under the appropriate conditions the simpler approach of Erdmann (1979a,b) can be used. One restriction on using approaches where P-R is assumed constant over the time increment is that the travel time between stations must be short (i.e., 1 to 3 hours) so that the constant P-R assumption is not violated.

3.7 REFERENCES


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