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GÖRSELLEŞTİRME BECERİLERİNİ GELİŞTİRMEYE YÖNELİK ÖGRETİM UYGULAMALARI PROBLEM CÖZMEDE GÖRSELLESTİRMENİN ETKİLİ KULLANIMINI DESTEKLER Mİ?

CAN TEACHING APPLICATIONS FOR DEVELOPING VISUALIZATION SKILLS PROMOTE EFFECTIVE VISUALIZATION USAGE IN PROBLEM SOLVING?

[a]Oya UYSAL KOĞ & [b]Neş'e BAŞER

[a]Dr., oyauysalkog@gmail.com

[a] Yrd.Doc.Dr., DEU Egitim Bilimleri Enstitusu, nese.baser@deu.edu.tr

Özet

Bu çalışmada, görselleştirme becerilerini geliştirmeye yönelik öğretim uygulamalarıyla yürütülen matematik derslerinin, öğrencilerin matematikte görselleştirmeyi kullanmaları ve etkili kullanmaları üzerindeki rolü araştırılmıştır. Kontrol gruplu ön test-son test modeline dayalı olan bu deneysel çalışmanın deney ve kontrol gruplarını 2010–2011 Eğitim-öğretim yılında İzmir´deki bir ilköğretim okulunun 8. sınıf öğrencileri oluşturmaktadır (N_{Deney}=21, $N_{kontrol}$=22). Deney grubunda "I.dereceden bir ve iki bilinmeyenli denklemler" konusunun işlenişi bilgisayar destekli görsel sunumlarla ve görselleştirme çalışmalarıyla yürütülmüştür. Öğrencilerin görselleştirmeyi kullanma durumları uygulama öncesi ve sonrası araştırmacılar tarafından hazırlanan "Matematikte Soyut Düşünme Testi"yle irdelenmiştir. Araştırma sonuçları görsel sunumlarla yürütülen matematik dersleri ve yapılan görselleştirme çalışmaları sonrasında problem çözmede görselleştirmenin kullanımında ve etkili biçimde kullanımında artış olduğunu göstermiştir.

Anahtar Sözcükler: Görselleştirme, matematiksel problem çözme, animasyonlar, lineer denklem sistemleri, matematik eğitimi.

Abstract

In this study, it has been investigated the role of teaching applications which intend to develop visualization skills of the students on visualization usage and effective visualization usage in problem solving. It is an experimental research based on an experimental pre-test post-test model. The experimental and control groups consist of the 8th grade (14-15-year-old) students of a secondary school in Izmir/ Turkey. The experiment group was formed of 21 students, while the control group was formed of 22 students. In the experimental group, teaching process of "systems of linear equations" subject was carried out by using computer-based visual representations and visualization practices. Before and after the teaching application and practices, the status of visualization usage and effective visualization usage of students was examined with "Abstract Thinking in Mathematics Test" which has been developed by the researchers. Results showed that to practice visualizing on routine problem solving effect the students´ preferences on using visual modes of thought and problem solving when they solve non-routine problems. According to posttest scores of experimental group, usage and effective usage of visualization in problem solving were increase.

Keywords*:* Visualization, mathematical problem solving, animations, systems of linear equations, mathematics education.

Introduction

The first reason making the mathematics difficult to comprehend is its "feature of being abstract". For this reason in elementary mathematics when the cases require abstract thinking, visual presentations and models are commonly used for the abstraction of concepts and problem solving. During the transition from arithmetic to algebra especially for the 8th graders in primary schools (15-year-olds), the algebraic approach is kept at the forefront for high school and college-level math teaching. Since the visual representations remained only in the early years of primary education, the role of visualization in the learning process through advanced levels is reduced, and the algebraic approach in math teaching is emphasized, and this resulted in perceiving the algebra education as a different branch from mathematics. Teaching with algebraic approach, enables the students to develop their abstract thinking skills. However, the absence of the expected level of abstract thinking skills can bring along some deficiencies in the use of conceptual and procedural knowledges. The structural deficiencies which can be described as a gap between arithmetic and algebra, can lead to difficulties for the internalization of abstract concepts and their relations with others during the transition from arithmetic to algebra. In other words, when it is got further away from the visual presentations, students from the 8th class of primary school to university level may have difficulties in visualizing the abstract mathematical concepts and establishing the necessary relations.

The studies for the development of Abstract Thinking Skills in Mathematics:

During the years covering the preschool period and concrete operational stage, Clements, Sarama ve DiBase (2004) have referred to "Using the words of numbers anywhere appropriately", "Playing with directions and maps", "Using materials", "Making a classification activity for any reason", "Talking over reasons and assumptions", "Helping to ask good questions", "Talking about talking, thinking about thinking" and "Concretization" as exemplars of some mathematical activities in order to develop abstract thinking. During the abstract operational stage (when the abstract thinking capacities of individuals get expanded) these activities should be carried up to advanced levels. The abstract thinking individual who has the flexibility of thoughts is expected to have developed behavior and skills to look at the events from many aspects, think with assumptions and probabilities apart from realities, think critical, solve many problems by mental processing, reasoning, arguing and combining few factors, make analysis and synthesis and understand the relationship between the object and the phenomenon. According to the standards of the math education program for this period, Schwarz, Dreyfus and Hershkowitz (2009) have referred to the necessity of "making research activities where the observation, hypothesis construction, generalization, and inspection stages can be carried out, creating meaningful situations which motivates for learning and persuading, creating activities associated with the prior knowledge of the individual, having them to think deeply about the knowledge they have on the subject" in the learning process.

Visualization in improving the success of mathematical problem solving

Mathematical problems by its nature involve many sentences connected with each other. Verbal problems sometimes occur in a long paragraph; therefore, a mathematical problem may seems like quite long, complex, abstract, and a pile of meaningless data for the students. In the study carried out by Joseph (2004) with 13-14 year old 56 secondary school students, the students who are unsuccessful in solving problems appeared to be unsatisfactory in grasping the problem, strategic knowledge, conversion of the problem in mathematical form and making the calculations. To overcome these obstacles in the problem solving process of the students, different approaches and strategies need to be used by the teachers and the students.

"Determining the hidden unknown of the problem", "expressing the relations of the sentences with each other by using the mathematical symbols" are important steps prior to the algebraic operations to find the unknown. These steps require abstraction in mathematics and should be gradually constructed in a proper manner. In mathematics, especially in problem solving, the most important benefit of the use of visualization is having the visualization convert the abstract into less abstract or to concrete, and supporting the abstract thinking and reasoning by this means. In other words, in a word problem, having meaningful drawings to ease the conversion of the sentences into correct mathematical expressions requires an advanced level of abstract thinking and reasoning. Arcavi (2003) qualifies visualization as a key component of reasoning (both conceptual and perceptual), problem solving, and proving. In this research, practicing visualization on routine problems and consolidation of the effective visualization usage with non-routine problems have been studied. At the end of this process, the answer is sought on whether the students perceive visualization as an alternative approach to algebraic approach and use it effectively while solving the non-routine problems. Before describing the details of this study, the literature review underlying the experimental approach is elaborated.

Literature Review:

We have referred to many remarkable studies in order to explain “the status of visualization in mathematics learning” from different perspectives.(e.g.; Eisenberg & Dreyfus, 1986; Bishop, 1989; Barwise ve Etchemendy (1991); Zimmermann & Cunningham (1991); Presmeg, 1992; English, 1997; Hitt (1998); Duval (1999); Arcavi, 2003; Presmeg, 2006).

Zimmermann & Cunningham (1991) described the mathematical visualization as the process of forming images (mentally, or with pencil paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding. This definition was blended with Hershkowitz et al. (1989)'s by Arcavi (2003). He gave a place the blend of "visualization" definition in his paper as follows:

"Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings."

The contribution of visual presentations to math teaching have been extensively studied in the 1990s. From the discussions on the current position of visualization in math teaching, the idea of "the visualization is not where it belongs in the learning process" has emerged. In their research named "Visual Information and Valid Reasoning", Barwise and Etchemendy (1991) mentioned that despite the importance of visual expressions in human cognitive activity is clear and unambiguous, they remained in the second place for the theory and practice of mathematics. They claimed that the visual expressions can not only be important as a shortcut or a pedagogical tool but as the reasonable elements of mathematical proofs. Similarly Dreyfus (1991) stated that the math educators appear to be aware of the importance of visual reasoning, but the students tend to avoid visual reasoning in exercises. He asserted a claim that the reason for this reluctance of the students is only because the teachers continue to emphasize the non-visual methods in teaching. Healy and Hoyles (1996) also pointed out this reluctance and emphasized the need for the creation of systems to support the students for establishing a link between mathematics and visualization. He pointed out the necessity of visualization to upgrade from the existing "learning support assistant" position to "a perfectly valid tool for learning and proving" in math teaching. In contrast to this opinion Duval (1999) emphasized that "from a learning point of view, visualization, the only relevant cognitive modality in mathematics, cannot be used as an immediate and obvious support for understanding."

Hitt (1998) mentioned that the teachers adopted the algorithmic-algebraic approach in math test books for concept teaching and encouraged to avoid taking visualization into account for problem solving. However when the visualization approach is used effectively, it also plays a supporting role for the creation of the algorithmic structure in the algebraic approach which is used to solve the problem.

Stylianou (2001) has the opinion that the image of "the reluctance against visualization" in college mathematics has started to change if compared with 10 years ago. Stylianou (2001) propounded that the perceptions of mathematicians and university students have changed on the use of visualization and stated that the students are willing to use visual representations however the studies on developing this skill are very limited in learning environments.

When approaching to the present day, the studies to determine the status of the current use of visualization and visual representations in learning mathematics and problem solving have revealed some important results. These results show that students who have become accustomed to the algebraic approach (which is given only in accordance with mathematical rules) have difficulties for the transfer of algebraic terms to visual representations or vice versa and the interpretation of different representations of an mathematical expression. (Rösken ve Rolka, 2006; Orhun, 2007; Zubieta ve Meza; 2008; Yenilmez ve Şan, 2008; Delice ve Sevimli, 2010). However, visual reasoning with the visualization provides convenience for the understanding of conceptual knowledge and making various generalizations in the process of abstraction. As Yenilmez ve Şan (2008) also expressed, visualization stimulates mental operations such as intuitive understanding of complex processes or establishing abstract relations. Based on this it can be said that the visualization contributes to the development of abstract thinking.

When reaching to the present day under the consideration of these studies, it is believed that there are not sufficient number of researches to demonstrate the power of visualization in learning mathematics, (as in this research), and the studies are carried out with a variety of class activities to demonstrate the power of visualization in mathematics education. (David & Tomaz 2011; Uysal Koğ & Başer; 2011).

In Turkey when the literature in the relevant field is analyzed, there exists the surveys conducted on the consideration of multiple representations of mathematical knowledge in verbal, numerical, graphical or algebraic way in the process of learning and their effective use during the process and the experimental studies. (Erbaş, 2005; Akkoç, 2005; Özmantar, Akkoç, Bingölbali, Demir ve Ergene, 2010; İpek ve Baran, 2011). Limited studies on the use of visual representations in math teaching were carried out with teacher candidates in high school level (Tekin ve Konyalıoğlu, 2009) and in undergraduate level (İpek, 2003; Konyalıoğlu, 2003; Türer ve Şengül, 2007).

Even though nowadays it is supported by the use of computers and other technological tools and devices, the essential tools of visualization are mind, pen and papers. Zimmermann & Cunningham (1991) recommended handling the field of mathematical visualization not only related to the computer based visualization but also to the non-computer based visualization. They mentioned that drawing a simple figure to represent a mathematical problem and using such figures as an aid in problem solving are fundamental visualization skills. Without such fundamental skills, it is impossible to use computer based visualization effectively.

In this experimental study, the subject of first-degree equations with one and two unknowns (system of linear equations) which exists in the eighth grade curriculum of primary education is taught in accordance with the visualization approach. The purpose of the design and use of visual materials used in the experimental process is to make the students understand that before the representation of the data in algebraic terms, visual representations can be used as intermediate step in word problems and to support the students to see the visualization as an alternative solution for other problems or complex situations and make this method work properly.

In this context the problem of the research is arranged in the form of "Does the use of visualization in mathematical problem solving studies increase a) the use of visualization in the process of problem-solving? b) the effective use of visualization?". However, in this research the accuracy of the assumption "if we compare the situation before and after the teaching process, the changes between the abstract thinking in math test scores of experiment and control group students indicate a statistically significant difference?" has been tested.

Method

Research Model

In this research a test model consisting of a pretest-posttest control group has been used. The subject of first-degree equations with one or two unknowns (systems of linear equations) is taught in accordance with the approach of visualization and computer aided visual presentations. Students belonging to the control group are taught the same subject under the supervision of their own teacher and in their own class environment without any changes. Before the practice, the control group teacher declared his instructional plan that is personally taught by himself verbally and that the students transfer notes of the relevant subject from the blackboard. Then lessons were finalized by solving various exercises about the new learned area. In light of this information, control group lessons can be described as Traditional Education Technique basis. The methods used in both groups are investigated for their effects on utilization level of visualization for students in problem solving and effective use of visualization.

Working group

The experimental and control groups consist of the 8th grade (14-15-year-old students) students of a secondary school in Izmir city during the 2010-2011 academic year. The experiment group was formed of 21 students ((N_{girls}=13, N_{boys}= 8), while the control group was formed of 22 students (N_{girls}=14, N_{boys}= 8).

Materials

Before the visual teaching application process, work sheets and computer aided visual materials were prepared by the researchers.

Worksheets:

Firstly,various types of word problems in the subject of *first-order equations with one or two unknowns* (systems of linear equations) were selected for the visual presentation. These problems which can be referred as routine problems exist in curriculum. Four types of problems with one unknown and four types of problems with two unknowns were selected. Then, three problems were composed from each of these problem types. Then a worksheet was prepared for each type of problem. An exemplar of this worksheet (shown in Apendıx.1) divides the teaching applications regarding problem solving into 3 sections which include the *in-class practices, individual practices and homework practices.*

Computer aided visual materials:

The solution of the first problem which is under the topic of "Let's solve together" was primarily solved as a draft on the paper by researchers visually. The drawings on the draft paper were then transferred to the computer with the use of animations. The data, connections between the sentences in the problem and connections between the data and the wanted were animated step by step, visually presented within this model. Macromedia Flash CS 5, Swish Max, iSpring and Power Point were used during the material development process (see the example in Online Resource 1). After developing, the animations were reviewed by the researchers, 3 mathematics teachers and 2 instructional technologies experts as effective learning tools in terms of both visual (eye-catching) and pedagogical (clear, in order, comprehensible, motivating) perspectives. Necessary changes were made according to expert views.

Procedure

The path is followed in this experimental study:

-1- *Teaching the subject via visualization approach "Let's solve it together"*

Particularly the subject of systems of linear equations that the students met for the first time, the animations were built based on inductive methods. In the animated presentation of the word problem stated in the first section of the work sheet (the section "Let's solve together"), The answers of the purposeful questions about the word problem such as "what's being asked to us?", "what do we already know?", "what do we need to know?", "what kind of algorithm can we use?" were emphasized with the support of visualization. In other words the word problem was modeled with the help of animations before the mathematical modeling. Modeling acted as a bridge between the verbal representation and the mathematical model of the problem. In the process of answering aforementioned questions the students were encouraged to reasoning, make comment on the solution. During discussion the animations were repeated step by step, forwards and backwards according to students' questions to resolve the misunderstanding. After discussion, students transferred this visual solution to the blank provided for the solution near the first question in work sheet.

-2- *Strengthening the use of visualization regarding the solution of routine problems: "Solve it yourself"*

After the solution of the first problem with the support of computer aided visualization, students worked individually on the solution of the second word problem which was in the section of "Solve it yourself". To be able to strengthen the usage of drawings, the students were asked to solve the second problem visually. It was emphasized that the visualization was used as a tool to solve the problem easier and therefore the visualizations may not be pictorial but symbolical. In other words, the students were advised against not wasting too much time on drawing but to make the drawings in a way that they can understand. After the practice of solving the word problem by constructing its visual model, the various solutions from students were discussed in class and the true solution was demonstrated by a voluntary student.

"At home" : The goal of the third problem in the work sheet (in the section of "At home…" was to reinforce the effective transformation the data from verbal to visual representation. This tripartite process was repeated for all eight work sheets which included different word problem types.

-3- *The solution of non-routine problems with the help of visualization approach*

During the lessons with the support of animations and visualization practices, the students were encouraged to transform 24 routine word problems to mathematical expressions by using drawings, visual representations. After this process on routine problems, to examine the development of visualization usage and effective visualization usage skill "Abstract Thinking in Math Test (ATMT)" was used.

Data collection

In this research, a multiple-choice test named "Abstract Thinking in Math Test" (ATMT) which has been developed by the researchers was used. The items of the ATMT has been prepared based on the abstractness of mathematical problems. The test includes the problems which are mostly non-routine and easier to solve with visualization. Up-to-date text books and test preparation books are analyzed before creating the items. Since the ATMT doesn't involve only one mathematical topic, items specifically focuses on higher order thinking skills and visualization usage. The students were expected to transfer "the solution", "the way of thinking" and "the followed algorithm" to the paper even though they solve the problem in their minds.

Development of "Abstract Thinking in Math Test" (ATMT)

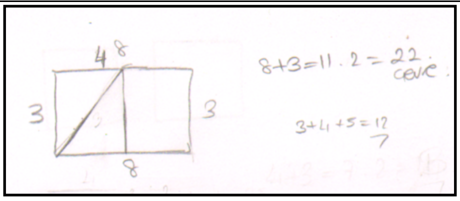
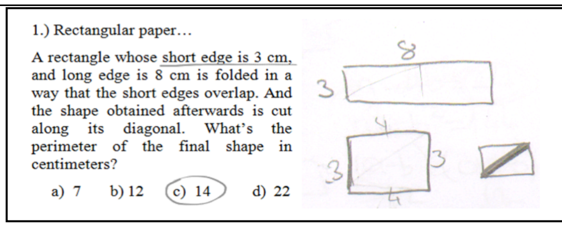
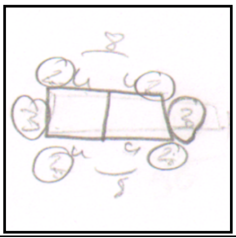
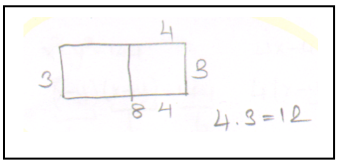
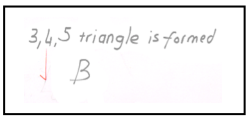
The draft version of the ATMT consists of 8 application, 6 analysis, 4 synthesis and 4 evaluation level items. After consulting 7 academicians working on mathematics education and 5 mathematics teachers, the corrections has been made on the items. The test has been developed by means of the item analysis via ITEMAN packaged software. Data were collected from 378 ninth-grade class students. For each item on the test p (item difficulty) and r (characteristic feature) values were calculated. As a result of the item analysis, 4 items were omitted whose characteristic feature values are below 0,29. Also the options of an item is rearranged and added to the test. The reliability (Cronbach Alpha) of the test with 18 items was calculated as 0,64. After the development of the ATMT, it was applied on 72 students within 2 month breaks and the reliability coefficient was calculated as 0,66. Maximum score available on ATMT is 100 points.

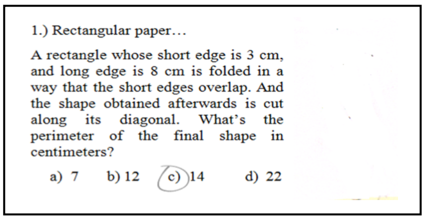
The evaluation of ATMT:

The Abstract Thinking in Math Test was evaluated via two different approaches: In the first approach, the scores gathered via choosing the correct responses in the test are calculated as (an achievement test) for **"overall test scores (OTS)"**. In the second evaluation approach was based on the level of visualization usage regarding the solutions of the problems. The scoring system with 5-point scale was designed to evaluate the test in terms of not only the response but also solution method. It was examined how often the students used visualization and whether they used it effectively or not.

In the second evaluation approach, the scoring system uses 5-point scale as "G+","G-",G/","+", "-" and the exemplar of evaluation is given in Table 1. The sample question regarding the evaluation in accordance with the criteria in Table 1 is the first question of the Abstract Thinking Test. The question is: *"A rectangle whose short edge is 3 cm, and long edge is 8 cm is folded in a way that the short edges overlap. And the shape obtained afterwards is cut along its diagonal. What's the perimeter of the final shape in centimeters? a) 7 b) 12 c) 14 d) 22".*

Table 1. The evaluation criteria of the answers given to the Abstract Thinking in Math Test (1.question sample)

Level of Visualization	Response category	Sample Response
G+	"Visual and correct" (Visualization used effectively, answered correctly.)	
G-	"Visual and incorrect" Visualization used effectively, not answered correctly.	
G/	Visualization not used effectively.	 
+	"nonvisual and correct" Visualization never used, answered correctly.	

-	"nonvisual and incorrect" Visualization never used, not answered correctly.	 1.) Rectangular paper… A rectangle whose short edge is 3 cm, and long edge is 8 cm is folded in a way that the short edges overlap. And the shape obtained afterwards is cut along its diagonal. What's the perimeter of the final shape in centimeters? a) 7 b) 12 c) 14 d) 22

As it can be observed in Table 1, "G+", "G-" and "G/" are the points for the answers which the students utilize visualization. Therefore the answers evaluated as "G+", "G-" and "G/" out of 18 questions constitute the **"visualization score (VS)"** belonging to that student. The questions graded with "G+" and "G-" are the questions which the students utilize the visualization effectively. And the score gathered via the points "G+" and "G-" constitutes the **"effective visualization score (EVS)"** of the student. The answers are evaluated by this scoring system and "visualization score (VS)" due to the use of visualization and "effective visualization score (EVS)" due to the solution oriented effective visualization usage were calculated for each student in pretest and posttest.

Table 2. An Example of two evaluation approaches of the responses given to the Abstract Thinking in Math Test

İtem	**Student A's Responses**	**1.Evaluation approach (Correct/ Incorrect)**	**2.Evaluation approach (G+, G-, G/, + , -)**	**2.a.Visualization usage ("G+"+"G-"+"G/")**	**2.b.Effective Visualization usage ("G+"+"G-")**
1	A	✓	G+	☺	☺
2	C	✓	G+	☺	☺
3	C	✗	G-	☺	☺
4	B	✓	G+	☺	☺
5	D	✓	+		
6	B	✓	G+	☺	☺
7	B	✗	G/	☺	
8	A	✗	G-	☺	☺
9	D	✓	+		
10	D	✗	-		
11	A	✓	G+	☺	☺
12	C	✗	-		
13	A	✗	-		

14	C	☑	G+	☺	☺
15	C	✗	G/	☺	
16	D	☑	+		
17	C	☑	+		
18	B	✗	G/	☺	
Scores		Sum of correct answers: 10 (10/18).100= 56 **OTS score= 56 points**		Sum of G+,G-,G/ = 11 (11/18).100= 61 **VS score = 61 points**	Sum of G+ and G- = 8 (8/18).100= 44 **EVS score = 44 points**

As it seen Table 2, Student A -with 10 correct choices- got 56 points as Overall test scores (OTS) (1.Evaluation approach as a multiple-choice test). When the same student's paper was assessed according to 2.evaluation approach, then he got 61 points as visualization scores (VS), due to his visual responses. And his effective visualization scores (EVS) was calculated as 44 points due to his definitive visual responses.

Analysis of Data

The data collected during this research were analyzed with the aid of the SPSS 19.0 packaged software. Shapiro Wilks normality test, Independent samples t-test, Wilcoxon signed ranks test and Mann Whitney U statistical tests are also used to analyze the data.

Findings

During the experimental process, we tried to make the students believe that "visualization can be a reinforcement for reasoning. And visual reasoning supports problem solving process in mathematics." Enhancing the *Volunteering status for drawings* (to visualize the problem) and *Problem solving success* was aim of this visual problem solving process. Therefore, in terms of the aim of the process, it makes sense to evaluate students' pretest and posttest papers according to two criteria: "Visualization usage" and "Correctness of answers".

The findings of this research is given respectively as a) *on item basis* and b) *on student basis.* On item basis evaluation, changing the status of "Visualization usage" and "Correctness of answers" were examined. On student basis evaluation, "visualization score (VS)" and "effective visualization score (EVS)" of experiment and control groups were compared.

a) Descriptive Analyses on item basis:

On item basis evaluation, we searched that if the visual learning process by scaffolding visualization practices enhanced the students' willing to visualization. The *percentages of visual and effective visual responses* are reported Table 3.

Table 3. Percentages of Pretest-Posttest *Visual Responses* belonging to Experimental Group Students

item no:		**Visual responses %**	**Effective Visual responses %**	**item no:**		**Visual responses %**	**Effective Visual responses %**
1	Pre.	91	67	**10**	Pre.	77	10
	Post.	*100*	*95*		*Post.*	*82*	*58*
2	Pre.	29	5	**11**	Pre.	82	34
	Post.	*81*	*48*		*Post.*	*81*	*76*
3	Pre.	24	19	**12**	Pre.	15	5
	Post.	*58*	*53*		*Post.*	*34*	*24*
4	Pre.	67	15	**13**	Pre.	53	29
	Post.	*95*	*38*		*Post.*	*86*	*57*
5	Pre.	29	5	**14**	Pre.	5	5
	Post.	*53*	*24*		*Post.*	*0*	*0*
6	Pre.	14	14	**15**	Pre.	43	24
	Post.	*25*	*20*		*Post.*	*72*	*67*
7	Pre.	19	19	**16**	Pre.	15	5
	Post.	*48*	*38*		*Post.*	*38*	*19*
8	Pre.	52	14	**17**	Pre.	0	0
	Post.	*77*	*67*		*Post.*	*0*	*0*
9	Pre.	71	47	**18**	Pre.	0	0
	Post.	*86*	*76*		*Post.*	*0*	*0*

As it seen in Table 3, percentages of visual and effective visual responses for all items increased after the experimental process out of two items (item 11 and item 14). But when we focus on the results of item 11, we can see the increase on effective visual responses (from %34 to % 76). In brief, students put more effort forth to make drawings, to solve the problems visual way after visual problem solving practices. More importantly, they used more powerful drawings. And when they didn't find it necessary then they didn't use it (As it seen Item 17 and 18- percentages of pre-post tests).

During the experimental process, the aim of animations and visualization practices was not to make the students "visualizer". More importantly is "to serve drawings to find out the correct answer". To obtain that if the visual learning process supports problem solving success, *percentage of visual/ non visual correct responses* are reported (see in Table 4).

Table 4. Percentages of Pretest-Posttest *Correct Responses* belonging to Experimental Group Students

Item no:		Visual & correct responses (G+ %)	Non-visual & correct responses (+ %)	Total correct responses	Item no:		Visual & correct responses (G+ %)	Non-visual & correct responses (+ %)	Total correct responses
1	Pre.	67	5	72	10	Pre.	5	5	10
	Post.	*81*	*0*	*81*		*Post.*	*48*	*10*	*58*
2	Pre.	5	48	53	11	Pre.	29	0	29
	Post.	*48*	*19*	*67*		*Post.*	*71*	*14*	*85*
3	Pre.	14	29	43	12	Pre.	5	48	53
	Post.	*43*	*33*	*76*		*Post.*	*24*	*57*	*81*
4	Pre.	5	10	15	13	Pre.	29	33	62
	Post.	*24*	*5*	*29*		*Post.*	*57*	*10*	*67*
5	Pre.	0	33	33	14	Pre.	5	67	72
	Post.	*19*	*43*	*62*		*Post.*	*0*	*90*	*90*
6	Pre.	14	71	85	15	Pre.	24	5	29
	Post.	*10*	*43*	*53*		*Post.*	*67*	*10*	*77*
7	Pre.	19	38	57	16	Pre.	5	33	38
	Post.	*33*	*29*	*62*		*Post.*	*19*	*48*	*67*
8	Pre.	14	19	33	17	Pre.	0	48	48
	Post.	*62*	*10*	*72*		*Post.*	*0*	*33*	*33*
9	Pre.	33	19	52	18	Pre.	0	71	71
	Post.	*76*	*14*	*90*		*Post.*	*0*	*67*	*67*

As it seen in Table 4, percentages of visual & correct responses for all items increased after the visual problem solving process out of two items (item 6 and item 14). As small but important detail, when we pay attention to the results of item 14, according to post test percentage of correct answer (90%), we can say that most of the students didn't find drawings necessary to solve the problem. Most of them thought that algebraic way is fit for this problem, preferred this way and succeed it.

b) Data Analyses on student basis:

Group Equivalency:

It was important to obtain initially whether control group students significantly differed from the students in the experimental group at the beginning. To determine the group equivalency, "overall test scores (OTS) - pretest data" of the ATMT were used. Before the comparison, w test for normality was made.

The analysis results of Shapiro-Wilks test showed that ATMT "overall test scores (OTS)" data are normally distributed ($W_{exp\text{-}pre}$= .948 , p= .315; $W_{exp\text{-}post}$= .955 , p= .418; $W_{control\text{-}pre}$=.923, p= .089; $W_{control\text{-}post}$=.915, p= .061). Therefore, independent samples t-test was used. As a result of Independent Samples t-test, it is observed that there is no statistically significant difference between the pretest scores of the experiment ($\overline{X}$: 56.66; sd: 16.75) and control ($\overline{X}$: 48.41; sd: 13.55) groups (z=0.82, p>.05). This situation indicates that both groups were equivalent in terms of overall test scores at the beginning of the procedure.

Research question (a):" Can teaching applications for developing visualization skills increase the use of visualization in the problem solving process?

After the comparison of the experimental group pretest-posttest visualization scores (VS) with the help of Wilcoxon signed ranks test, it is observed that there is statistically significant difference between the pretest ($\overline{X}_{exp\text{-}pre}$: 37.76; sd: 19.65) and posttest ($\overline{X}_{exp\text{-}post}$: 56.19; sd: 18.34) scores (z=3.57, p<.05). And as it seems this difference in favor of the posttest visualization scores (VS_{Post}). According to the results, it can be said that visual teaching techniques and visualization practices can provide an increase in the use of visualization for the students.

Research question (b):" Can teaching applications for developing visualization skills increase the effective use of visualization in the problem solving process?

After the comparison of the pretest-posttest effective visualization scores (EVS) of experiment group students with the help of Wilcoxon signed ranks test, it is observed that there is statistically significant difference between the pretest ($\overline{X}$: 17.52; sd: 14.53) and posttest ($\overline{X}$: 42.24; sd: 22.1) scores (z=3.83, p<.05). This difference in favor of the posttest effective visualization scores (EVS_{Post}). According to this, we can say that visual teaching techniques and visualization practices can provide an increase in correct and effective use of visualization for the students.

The question of ***"Did the control group students have similar results, suggesting that the significant changes cited above are not due to the intervention, but due to the mathematics course?*** is also significant in terms of experimental process. To find out the answer of this question, control group students' "use of visualization scores" (VS) were also compared.

After the examination of the pretest-posttest visualization points of control group students (VS) with the help of Wilcoxon signed ranks test, it is observed that there is no statistically significant difference between the pretest ($\overline{X}$: 28.27; sd: 10.98) and posttest ($\overline{X}$: 34.09; sd: 32.51) scores (z=.99, p>.05). Similarly, after the comparison of the pretest-posttest effective visualization scores (EVS) of control group students with the help of Wilcoxon signed ranks test, it is observed that there is no statistically significant difference between the pretest ($\overline{X}$: 11.77; sd: 10.38) and posttest ($\overline{X}$: 26.41; sd: 31.44) scores (z=1.83, p>.05).

To find out the answer of the question:"***Is there a pretest-posttest difference regarding the use of visualization and effective use of visualization between the teaching group which utilizes the visual presentations and the teaching group with traditional teaching approaches?***", visualization scores (VS) and effective visualization scores (EVS) of experiment and control groups were compared with Mann Whitney U test. The results are provided in Table 5.

Table 5**.** The u-test results regarding the comparison of the pretest-posttest visualization points (VS) and effective visualization points (EVS) for the students involved in experiment and control groups

Variable	**Measu.**	**Group**	**n**	**Mean Rank**	**Sum of Ranks**	**U**	**p**
Use of Visualization (VS)	pretest	Exp.	21	25.52	536.00	157.00	.068
		Control	22	18.64	410.00		
	posttest	Exp.	21	27.29	573.00	120.00	.007*
		Control	22	16.95	373.00		

Effective use of Visualization (EVS)	pretest	Exp.	21	24.45	513.50	179.50	.205
		Control	22	19.66	432.50		
	posttest	Exp.	21	26.81	563.00	130.00	.014*
		Control	22	17.41	383.00		

* p<.05

After the evaluation of the pretest visualization scores (VS) of experiment ($\overline{X}_{exp\text{-}pre\text{-}VS}$: 37.76; sd: 19.65) and control ($\overline{X}_{control\text{-}pre\text{-}VS}$: 28.27; sd: 10.98) groups, it is observed that there is no statistically significant difference (U=157.00, p>.05). Similar to this, there is no statistically significant difference between the pretest effective visualization scores (EVS) of the two groups ($\overline{X}_{exp\text{-}pre\text{-}EVS}$:17.52;sd:14.53), ($\overline{X}_{control\text{-}pre\text{-}EVS}$:11.77;sd:10.38), (U=179.50,p>.05). But the comparison of the posttest visualization scores (VS) of experiment ($\overline{X}_{exp\text{-}post\text{-}VS}$: 56.19; sd: 18.34) and control ($\overline{X}_{control\text{-}post\text{-}VS}$: 34.,09; sd: 32.51) groups, it is observed that there is statistically significant difference in the advantage of experiment group (U=120.00, *p<.05). Concordantly after the comparison of the posttest effective visualization scores (EVS) of examination ($\overline{X}_{exp\text{-}post\text{-}EVS}$: 42.24; sd: 22.10) and control ($\overline{X}_{control\text{-}post\text{-}EVS}$: 26.41; sd: 31.44) groups, it is observed that there is statistically significant difference in the advantage of experiment group (U=130.00, *p<.05). According to these statistical results, the teaching method which utilizes the visual teaching techniques regarding the effective use of visualization for the students is more efficient than the method with traditional teaching approaches.

Results and Discussion

The main idea of this research is to increase the use of visualization and strengthening the effective use of visualization with problem solving activities. Therefore the solutions of routine problems were made via computer aided visual presentations. Separate and common visual problem solving practices were carried out. The tendencies to use drawings and the status of visualization usage were evaluated by the Abstract Thinking in Math Test before and after the experimental process.

The item-based evaluations regarding the Abstract Thinking in Math Test which consists of non-routine problems show that visual mode of teaching and thinking and problem solving practices with drawings encourage the students to use the visualization and improve their performances about using visualization effectively. While students in both groups were initially using algebraic methods to solve the problems before the process, it is observed that rigorous visualizations were made both for understanding the problem and simplifying the solution in the final evaluation results of the students who took part in experimental study.

Harel's study on teaching linear algebra (1989) has revealed that the visualization of concepts can help to remove learning disabilities and overcome the obstacles successfully. Clements and Compo stated that the visual fraction activities enable us to keep the concepts strongly in mind in their study of the effectivity of visual approach conducted by 21 elementary school students (Adapted by İpek, 2003). Studies of Rösken-Rolka (2006), İpek (2003), Konyalıoğlu (2003), Afamasaga-Fuata´I (2004) and Zubieta-Meza (2008) also resulted in a conclusion that the visualization approaches significantly contribute to the mathematical problem solving skills of the students. The results of this study indicates two significant points in terms of mathematics lessons:

Is Visualization an aim or a tool?

We should clarify that during the visual problem solving process, we were instilling to the students the importance of drawings and visual reasoning in problem solving. We aimed to provide the students a learning environment where they can improve their own drawings, visual view and visual reasoning on various problems. Namely, visualization was at the exact center of our aim. While we were doing these, we emphasized that main aim is successfully problem solving, not visualization usage. At the end of problem solving process which was carried out this aspect, students used drawings mostly when they required.

We can't generalize the "visualization support" for "all mathematical situations" and "all individuals":

In this study, it was worked on equations and systems of linear algebraic equations. During the process, it was focused on development of students' visualization and modeling usage only this subject. In terms of mathematical subjects, the answer of the question: "if visualization approach is useful for teaching other mathematical subject?" is discussible.

Because drawings are made on the purpose of clarify, simplify the mathematical concept or situation. It varies from person to person. Additionally, "Mathematical problem solving" is independently substantial area. Visualization can be the best way -for only required problem posing-, not all of them. When problem posing is not clear enough to solve, then drawings can come into prominence. Otherwise it can be waste of time. However visual solution methods were practiced during the experimental process, the final decision of choosing the solution method referred to the students. Results show that some problems were solved in algebraic way successfully by the students. At the end of the process, students had enough knowledge about alternative solution methods and they made more conscious decisions than at the beginning.

On the other hand it can't be said that the individuals choosing to use the visualization approach have abstract intelligence, and the others have analytical, algebraic intelligence. Presmeg (2006) also stated that the individuals who have abstract intelligence may not refer to visual methods time to time. As an example the experimental work of Lawrie ve Kay (2001) resulted in a conclusion that the students use visualization when solving verbal or difficult problems and choose to benefit from the non-visual methods for the problems which can be perceived easier.

Most of mathematical concepts (from basic to advanced level) have an image, indeed. However they seem as abstract, they can be described more perceptibly way. Rösken & Rolka (2006) stated in their study, the students have difficulties on visual demonstration of mathematical concepts which are given whilst working on specific problems. The number of the students that could formulate the given visual expression of a mathematical concept are very few and the success rate is very low since the students try to solve problems via algorithmic methods instead of visualization approach. Starting from this point of view they stated that the students can't correlate the visualization with mathematics and the visualization is difficult but important for them. In a similar way, Zubieta and Meza (2008) mentioned that the basic theorem of Analysis emphasizes the algorithmic way apart from the important connections between the derivative and integral topics.

Foremost is propounding the connections between mathematical concepts and their images with various visualization activities. For this reason, utilizing the visual presentations as a "middle step" between algebra and arithmetic in problem solving process may smooth the way for making various generalizations and abstraction. The idea of "using drawings as a middle step" can be presented to the students in a strengthening manner just as teaching algebraic approaches. The problem solving exercises in the class can build learning environments for strengthening the use of visualization.

Students should define the most suitable method for themselves and develop an inner system according to their decisions. They should witness different outer systems and join different visualization exercises initially. When viewed from this aspect, the use of visualization in the classrooms for math education should be popularized and teachers should make the students understand that this useful, alternative method also exists with the help of various exercises.

References

Afamasaga-Fuata´I, K. (2004). Concept Maps and Vee Diagrams in Undergraduate Mathematics Problem Solving. ICME-10, Mexico. *10th International Congress on Mathematical Education.*

Akkoç, H. (2005). Fonksiyon kavramının anlaşılması: Çoğul temsiller ve Tanımsal Özellikler, *Eğitim Araştırmaları Dergisi*, Yıl 5, Sayı 20, s. 14 – 2.

Barwise, J., Etchemendy, J. (1991): Visual information and valid reasoning. In W. Zimmermann & S. Cunningham (Eds.). *Visualization in Teaching and Learning Mathematics*, (pp. 1-8). Mathematical Association of America, Washington, DC.

Dede, Y. (2004). Öğrencilerin Cebirsel Sözel Problemleri Denklem Olarak Yazarken Kullandıkları Stratejilerin Belirlenmesi. Matematikçiler Derneği Bilim Köşesi. www.matder.org.tr.

Delice ve Sevimli (2010). Matematik Öğretmeni Adaylarının Belirli İntegral Konusunda Kullanılan Temsiller ile İşlemsel ve Kavramsal Bilgi Düzeyleri. *Gaziantep Üniversitesi Sosyal Bilimler Dergisi*, 9(3):581 -605.

Dreyfus, T. (1991). On the status of visual reasoning in mathematics and mathematics education. In F. Furinghetti (Ed.), *Proceedings of the 15th PME International Conference*, 1, 33-48.

Eisenberg, T., Dreyfus, T. (1991). On the reluctance to visualize in mathematics. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in teaching and learning mathematics* (pp. 26–37). Washington, DC: Mathematical Association of America.

Erbaş, K. (2005). Çoklu Gösterimlerle Problem Çözme ve Teknolojinin Rolü. *The Turkish Online Journal of Educational Technology* , volume 4 Issue 4 Article 12.

Hitt, F. (Ed.) (2002). "Representations and mathematics visualization*."(Paper presented in this Working Group of PME-NA, 1998-2002)*. Mexico City: Cinvestav– IPN.

İpek, A.S. (2003). "Kompleks Sayılarla İlgili Kavramların Anlaşılmasında Görselleştirme Yaklaşımının Etkinliğinin İncelenmesi." Yayınlanmamış Doktora Tezi, Atatürk Üniversitesi, Fen Bilimleri Enstitüsü, Erzurum.

İpek, A.S., Baran, D. (2011). "İlköğretim Matematik Öğretmen Adaylarının Teknoloji Destekli Temsillerle İlgili Düşünceleri." *5th International Computer & Instructional Technologies Symposium*, 22-24 September 2011, Fırat Universitesi, Elazığ.

Karasar, N. (2002). *Bilimsel Araştırma Yöntemi*. Ankara: Nobel Yayın Dağıtım.

Konyalıoğlu, A. C., (2003). "Üniversite Düzeyinde Vektör Uzaylari Konusundaki Kavramların Anlaşılmasında Görselleştirme Yaklaşımının Etkinliğinin incelenmesi." Yayımlanmamış Doktora Tezi, Atatürk Üniversitesi Fen Bilimleri Enstıtüsü, Erzurum.

Lawrie, T., Kay, R. (2001). Relationship Between Visual and Non Visual Solution Methods and Difficulty in Elementary Mathematics. *The Journal of Educational Research*, Volume 94, Issue 4.

Orhun, N. (2007). Kesir İşlemlerinde Formal Aritmetik ve Görselleştirme Arasındaki Bilişsel Boşluk. *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, 8(14), 99-111.

Özdemir, M. E, Duru, A. & Akgün, L. (2005). İki ve Üç Boyutlu Düşünme: İki ve Üç Boyutlu Geometriksel Şekillerle Bazı Özdeşliklerin Görselleştirilmesi. *Kastamonu Eğitim Dergisi*, 13(2), 527-540.

Özmantar M. F., Akkoç, H., Bingölbali, E. , Demir S. & Ergene B. (2010). Pre-Service Mathematics Teachers' Use of Multiple Representations in Technology-Rich Environments, *Eurasia Journal of Mathematics*, Science & Technology Education, 2010, 6(1), 19-36.

Presmeg, N. C. (2006). Research On Visualization in Learning And Teaching Mathematics: Emergence From Psychology. In A. Gutiérrez and P. Boero (Eds.), *Handbook of Research on The Psychology of Mathematics Education*. Dordrecht: Sense Publishers.

Rösken, B. & Rolka, K. (2006). A picture is worth a 1000 words - the role of visualization in mathematics learning. In J. Novotná, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 441-448). Prague, Czech Republic: PME.

Stylianou, D. (2001). On The Reluctance To Visualize in Mathematics: Is The Picture Changing? In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th PME International Conference*, 4, 225-232.

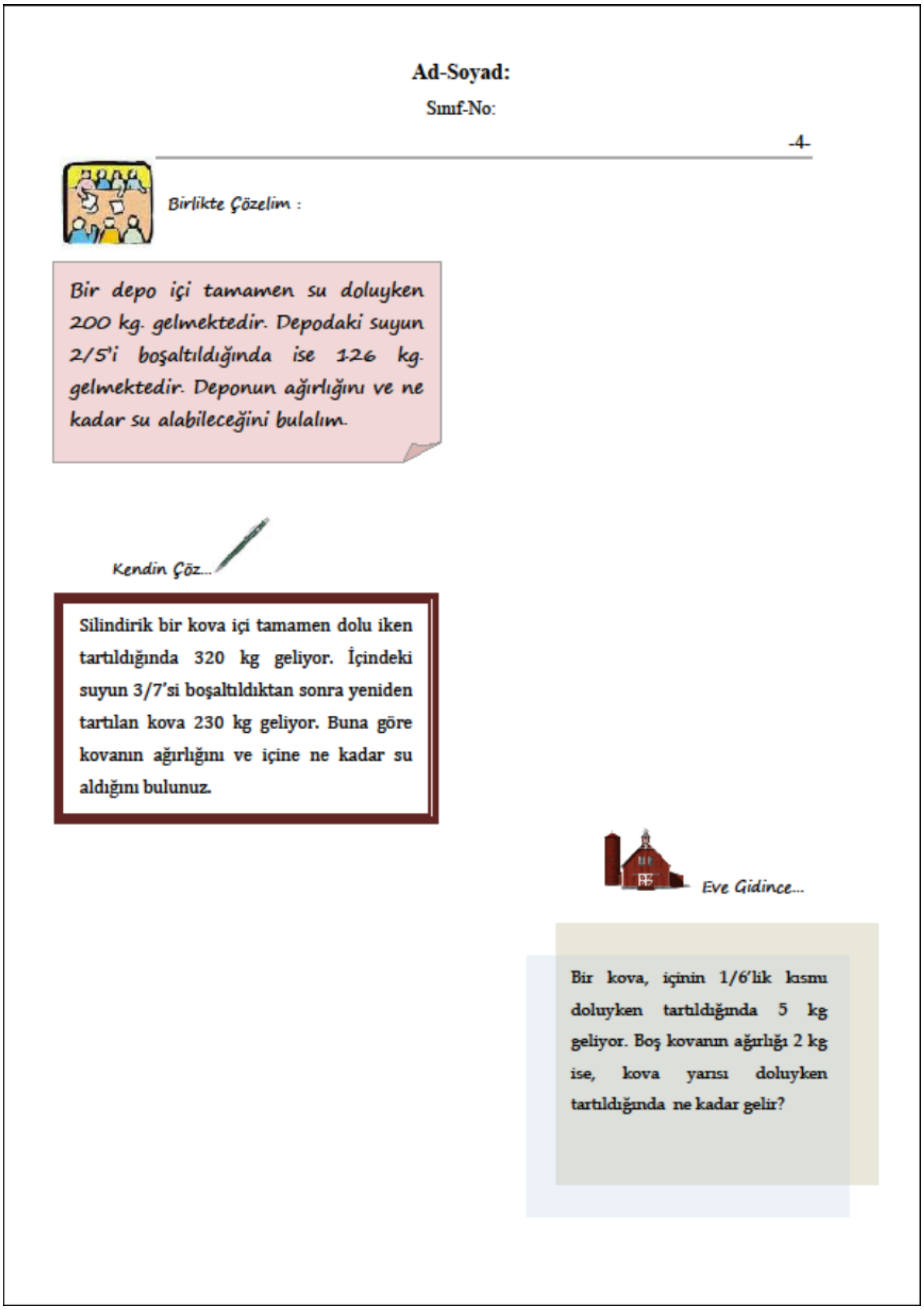
Tekin, B., Konyalıoğlu, A.C.. (2009). Dönüşüm ve Ters Dönüşüm Formüllerinin İspatlarının Ortaöğretim Düzeyinde Görselleştirilmesi, *Mat Der 8. Matematik Sempozyumu*

Türer, C., Şengül, S. (2007). "Öğretmen Adaylarının Matematik Problemlerini Problem Çözme Sürecinde Görselleştirme ve Basitleştirme Becerilerinin Tespiti." *XVI. Ulusal Eğitim Bilimleri Kongresi, 5-7 Eylül,* Gaziosmanpaşa Üniversitesi Eğitim Fakültesi, Tokat.

Uysal Kog, O. (2012). The Effect of Teaching Mathematics with Visualization Approach on Students' Cognitive and Affective Development, Doctorate Thesis, Dokuz Eylül University, Institute of Educational Sciences, İzmir/Turkey.

Yenilmez, K., Şan, İ. (2008). Dokuzuncu Sınıf Öğrencilerinin Matematiksel Kavramların Görsel Modellerini Tanıma Düzeyleri, *E-Journal of New World Sciences Academy*, cilt:3, sayı:3, sayfa:409-418

Zubieta, G., Meza, R. (2008). "Visualization in the teaching/learning of Calculus." *ICME-11, Mexico. 11th International Congress on Mathematical Education.*



Ad-Soyad:

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Birlikte Çözelim :

Bir depo içi tamamen su doluyken 200 kg. gelmektedir. Depodaki suyun 2/5'i boşaltıldığında ise 126 kg. gelmektedir. Deponun ağırlığını ve ne kadar su alabileceğini bulalım.

Kendin Çöz...

Silindirik bir kova içi tamamen dolu iken tartıldığında 320 kg geliyor. İçindeki suyun 3/7'si boşaltıldıktan sonra yeniden tartılan kova 230 kg geliyor. Buna göre kovanın ağırlığını ve içine ne kadar su aldığını bulunuz.

Eve Gidince...

Bir kova, içinin 1/6'lık kısmı doluyken tartıldığında 5 kg geliyor. Boş kovanın ağırlığı 2 kg ise, kova yarısı doluyken tartıldığında ne kadar gelir?

Fig.1 An exemplar of worksheets