## Supplementary material for the article "Alpha-cut based fuzzy cognitive maps with applications in decision-making"

DATA AND RESULTS RELATED TO:

## RADIOTHERAPY TREATMENT PLANNING

Table S. 4 Strengths of causal relationships in radiotherapy planning model

|  |  | Strength of causal relationship |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Causal <br> connection | Sign | Crisp | T1 fuzzy | IT2 fuzzy |
| $\mathrm{C}_{1} \rightarrow \mathrm{C}_{7}$ | + | 0.5 | $(0.4,0.5,0.6)$ | $((0.4,0.5,0.6 ; 1),(0.45,0.5,0.55 ; 0.8))$ |
| $\mathrm{C}_{2} \rightarrow \mathrm{C}_{1}$ | + | 0.3 | $(0.05,0.3,0.55)$ | $((0.05,0.3,0.55 ; 1),(0.1,0.3,0.5 ; 0.8))$ |
| $\mathrm{C}_{2} \rightarrow \mathrm{C}_{7}$ | + | 0.6 | $(0.5,0.6,0.7)$ | $((0.5,0.6,0.7 ; 1),(0.55,0.6,0.65 ; 0.8))$ |
| $\mathrm{C}_{3} \rightarrow \mathrm{C}_{2}$ | - | 0.3 | $(0.2,0.3,0.4)$ | $((0.2,0.3,0.4 ; 1),(0.25,0.3,0.35 ; 0.8))$ |
| $\mathrm{C}_{3} \rightarrow \mathrm{C}_{7}$ | - | 0.25 | $(0.15,0.25,0.35)$ | $((0.15,0.25,0.35 ; 1),(0.2,0.25,0.30 ; 0.8))$ |
| $\mathrm{C}_{4} \rightarrow \mathrm{C}_{5}$ | - | 0.4 | $(0.3,0.4,0.5)$ | $((0.3,0.4,0.5 ; 1),(0.35,0.4,0.45 ; 0.8))$ |
| $\mathrm{C}_{4} \rightarrow \mathrm{C}_{7}$ | - | 0.3 | $(0.2,0.3,0.4)$ | $((0.2,0.3,0.4 ; 1),(0.25,0.3,0.35 ; 0.8))$ |
| $\mathrm{C}_{5} \rightarrow \mathrm{C}_{4}$ | - | 0.3 | $(0.2,0.3,0.4)$ | $((0.2,0.3,0.4 ; 1),(0.25,0.3,0.35 ; 0.8))$ |
| $\mathrm{C}_{5} \rightarrow \mathrm{C}_{7}$ | + | 0.6 | $(0.5,0.6,0.7)$ | $((0.5,0.6,0.7 ; 1),(0.55,0.6,0.65 ; 0.8))$ |
| $\mathrm{C}_{6} \rightarrow \mathrm{C}_{2}$ | + | 0.55 | $(0.3,0.55,0.8)$ | $((0.3,0.55,0.8 ; 1),(0.35,0.55,0.75 ; 0.8))$ |
| $\mathrm{C}_{6} \rightarrow \mathrm{C}_{7}$ | + | 0.5 | $(0.25,0.5,0.75)$ | $((0.25,0.5,0.75 ; 1),(0.3,0.5,0.7 ; 0.8))$ |
| $\mathrm{C}_{7} \rightarrow \mathrm{C}_{1}$ | + | 0.3 | $(0.05,0.3,0.55)$ | $((0.05,0.3,0.55 ; 1),(0.1,0.3,0.5 ; 0.8))$ |
| $\mathrm{C}_{7} \rightarrow \mathrm{C}_{2}$ | + | 0.7 | $(0.45,0.7,0.95)$ | $((0.45,0.7,0.95 ; 1),(0.5,0.7,0.9 ; 0.8))$ |
| $\mathrm{C}_{7} \rightarrow \mathrm{C}_{5}$ | + | 0.55 | $(0.45,0.55,0.65)$ | $((0.45,0.55,0.65 ; 1),(0.5,0.55,0.6 ; 0.8))$ |

Table S. 5 Initial concept values in radiotherapy planning model

|  | Scenario 1 |  |  | Scenario 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Concept | Crisp | T1 fuzzy | IT2 fuzzy | Crisp | T1 fuzzy | IT2 fuzzy |
| C1 | 0.75 | (0.75,0.75, 0.75 ) | $\begin{aligned} & \hline(0.75,0.75,0.75 ; 1) \\ & (0.75,0.75,0.75 ; 1) \end{aligned}$ | 0.80 | (0.6,0.8,1) | $\begin{gathered} ((0.6,0.8,1 ; 1), \\ (0.65,0.8,0.95 ; 0.8)) \end{gathered}$ |
| C2 | 0.8 | (0.80,0.80,0.80) | $\begin{aligned} & (0.80,0.80,0.80 ; 1) \\ & (0.80,0.80,0.80 ; 1) \end{aligned}$ | 0.85 | (0.7,0.85,1) | $\begin{gathered} ((0.7,0.85,1 ; 1), \\ (0.75,0.85,0.95 ; 0.8)) \end{gathered}$ |
| C3 | 0.3 | (0.3,0.3,0.3) | $\begin{gathered} (0.3,0.3,0.3 ; 1) \\ (0.3,0.3,0.3 ; 1) \end{gathered}$ | 0.25 | $(0,0.25,0.5)$ | $\begin{gathered} ((0,0.25,0.5 ; 1), \\ (0.05,0.25,0.45 ; 0.8)) \end{gathered}$ |
| C4 | 0.6 | (0.6,0.6,0.6) | $\begin{gathered} (0.6,0.6,0.6 ; 1) \\ (0.6,0.6,0.6 ; 1) \end{gathered}$ | 0.45 | (0.3,0.45,0.6) | $\begin{gathered} ((0.3,0.45,0.6 ; 1), \\ (0.35,0.45,0.55 ; 0.8)) \end{gathered}$ |
| C5 | 0.7 | (0.7,0.7,0.7) | $\begin{aligned} & (0.7,0.7,0.7 ; 1) \\ & (0.7,0.7,0.7 ; 1) \end{aligned}$ | 0.60 | (0.5,0.6,0.7) | $\begin{gathered} ((0.5,0.6,0.7 ; 1), \\ (0.55,0.6,0.65 ; 0.8)) \end{gathered}$ |
| C6 | 0.5 | (0.5,0.5,0.5) | $\begin{aligned} & (0.5,0.5,0.5 ; 1) \\ & (0.5,0.5,0.5 ; 1) \end{aligned}$ | 0.55 | (0.4,0.55,0.7) | $\begin{gathered} ((0.4,0.55,0.7 ; 1), \\ (0.45,0.55,0.65 ; 0.8)) \end{gathered}$ |
| C7 | 0.65 | (0.65,0.65,0.65) | $\begin{aligned} & (0.65,0.65,0.65 ; 1) \\ & (0.65,0.65,0.65 ; 1) \\ & \hline \end{aligned}$ | 0.25 | (0,0.25,0.5) | $\begin{gathered} ((0,0.25,0.5 ; 1), \\ (0.25,0.25,0.45 ; 0.8)) \\ \hline \end{gathered}$ |

Table S. 6 Simulation results of radiotherapy planning model with hyperbolic tangent function

| $\lambda$ | Concept | Scenario 1 |  |  |  | Scenario 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Crisp | T1 fuzzy | IT2 fuzzy | Crisp | T1 fuzzy | IT2 fuzzy |
| 1.0 | C1 | 0.901 | (0.598,0.901,0.967) | $\begin{gathered} ((0.598,0.901,0.967 ; 1), \\ (0.719,0.901,0.959 ; 0.8)) \end{gathered}$ | 0.901 | (0.598,0.901,0.967) | $\begin{aligned} & ((0.598,0.901,0.967 ; 1), \\ & (0.719,0.901,0.959 ; 0.8)) \end{aligned}$ |
|  | C2 | 0.926 | (0.861,0.926,0.958) | $\begin{aligned} & ((0.861,0.926,0.958 ; 1), \\ & (0.879,0.926,0.953 ; 0.8)) \end{aligned}$ | 0.927 | (0.861,0.926,0.959) | $\begin{gathered} ((0.861,0.926,0.959 ; 1), \\ (0.879,0.926,0.953 ; 0.8)) \end{gathered}$ |
|  | C3 | 0.031 | (0.031,0.031,0.031) | $\begin{aligned} & ((0.031,0.031,0.031 ; 1), \\ & (0.031,0.031,0.031 ; 0.8)) \end{aligned}$ | 0.000 | (0,0.031,0.031) | $\begin{gathered} ((0,0.031,0.031 ; 1), \\ (0.026,0.031,0.031 ; 0.8)) \end{gathered}$ |
|  | C4 | -0.792 | (-0.842,-0.792,-0.716) | $\begin{aligned} & ((-0.842,-0.792,-0.716 ; 1), \\ & (-0.819,-0.792,-0.758 ; 0.8)) \end{aligned}$ | -0.79 | .842,-0.792,-0.716) | $\begin{aligned} & ((-0.842,-0.792,-0.716 ; 1), \\ & (-0.819,-0.792,-0.758 ; 0.8)) \end{aligned}$ |
|  | C5 | 0.948 | (0.917,0.948,0.966) | $\begin{gathered} ((0.917,0.948,0.966 ; 1), \\ (0.935,0.948,0.958 ; 0.8)) \end{gathered}$ | 0.948 | (0.917,0.948,0.966) | $\begin{gathered} ((0.917,0.948,0.966 ; 1), \\ (0.935,0.948,0.958 ; 0.8)) \end{gathered}$ |
|  | C6 | 0.031 | (0.031,0.031,0.031) | $\begin{aligned} & ((0.031,0.031,0.031 ; 1), \\ & (0.031,0.031,0.031 ; 0.8)) \end{aligned}$ | 0.031 | (0.031, $0.031,0.031)$ | $\begin{gathered} ((0.031,0.031,0.031 ; 1), \\ (0.031,0.031,0.031 ; 0.8)) \end{gathered}$ |
|  | C7 | 0.993 | (0.978,0.993,0.997) | $\begin{gathered} ((0.978,0.993,0.997 ; 1), \\ (0.987,0.993,0.996 ; 0.8)) \end{gathered}$ | 0.993 | (0.978,0.993,0.997) | $\begin{gathered} ((0.978,0.993,0.997 ; 1), \\ (0.987,0.993,0.996 ; 0.8)) \end{gathered}$ |
| 3.0 | C1 | 1.000 | (0.997,1,1) | $\begin{aligned} & ((0.997,1,1 ; 1), \\ & (0.998,1,1 ; 0.8)) \end{aligned}$ | 1.000 | (0.997,1,1) | ((0.997,1,1;1),(0.998,1,1;0.8)) |
|  | C2 | 1.000 | (0.999,1,1) | ((0.999,1,1;1),(1,1,1;0.8)) | 1.000 | $(0.999,1,1)$ | ((0.999,1,1;1),(1,1,1;0.8)) |
|  | C3 | 0.995 | (0.995,0.995,0.995) | $\begin{gathered} ((0.995,0.995,0.995 ; 1), \\ (0.995,0.995,0.995 ; 0.8)) \end{gathered}$ | 0.000 | (0,0.995,0.995) | $\begin{gathered} ((0,0.995,0.995 ; 1), \\ (0.995,0.995,0.995 ; 0.8)) \end{gathered}$ |
|  | C4 | 0.963 | (0.911,0.963,0.982) | $\begin{aligned} & ((0.911,0.963,0.982 ; 1), \\ & (0.945,0.963,0.975 ; 0.8)) \end{aligned}$ | 0.963 | (-1,0.963,0.982) | $\begin{gathered} ((-1,0.963,0.982 ; 1), \\ (-0.999,0.963,0.975 ; 0.8)) \end{gathered}$ |
|  | C5 | 0.998 | (0.993,0.998,0.999) | $\begin{aligned} & ((0.993,0.998,0.999 ; 1), \\ & (0.997,0.998,0.999 ; 0.8)) \end{aligned}$ | 0.998 | (0.993,0.998,1) | $\begin{gathered} ((0.993,0.998,1 ; 1) \\ (0.997,0.998,1 ; 0.8)) \end{gathered}$ |
|  | C6 | 0.995 | (0.995,0.995,0.995) | $\begin{aligned} & ((0.995,0.995,0.995 ; 1), \\ & (0.995,0.995,0.995 ; 0.8)) \end{aligned}$ | 0.995 | (0.995,0.995,0.995) | $\begin{aligned} & ((0.995,0.995,0.995 ; 1), \\ & (0.995,0.995,0.995 ; 0.8)) \end{aligned}$ |
|  | C7 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) |
| 5.0 | C1 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) |
|  | C2 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) |
|  | C3 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 0.000 | $(0,1,1)$ | ((0,1,1;1),(1,1,1;0.8)) |
|  | C4 | 0.998 | (0.995,0.998,0.999) | $\begin{gathered} ((0.995,0.998,0.999 ; 1), \\ (0.997,0.998,0.999 ; 0.8)) \end{gathered}$ | 0.998 | (-1,0.998,0.999) | $\begin{gathered} ((-1,0.998,0.999 ; 1), \\ (0.997,0.998,0.999 ; 0.8)) \end{gathered}$ |
|  | C5 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) |
|  | C6 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) |
|  | C7 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) |

Table S. 7 Simulation results of radiotherapy planning model with sigmoid function

|  | Scenario 1 |  |  |  |  | Scenario 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\lambda$ | Concept | Crisp | T1 fuzzy | IT2 fuzzy | Crisp | T1 fuzzy | IT2 fuzzy |
| 1.0 | C1 | 0.787 | (0.681,0.787,0.868) | $\begin{gathered} ((0.681,0.787,0.868 ; 1), \\ (0.704,0.787,0.854 ; 0.8)) \end{gathered}$ | 0.787 | (0.681,0.787,0.868) | $\begin{gathered} ((0.681,0.787,0.868 ; 1), \\ (0.704,0.787,0.854 ; 0.8)) \end{gathered}$ |
|  | C2 | 0.837 | (0.739,0.837,0.9) | $\begin{gathered} ((0.739,0.837,0.9 ; 1), \\ (0.768,0.837,0.887 ; 0.8)) \end{gathered}$ | 0.837 | (0.739,0.837,0.9) | $\begin{gathered} ((0.739,0.837,0.9 ; 1), \\ (0.768,0.837,0.887 ; 0.8)) \end{gathered}$ |
|  | C3 | 0.659 | (0.659,0.659,0.659) | $\begin{aligned} & ((0.659,0.659,0.659 ; 1), \\ & (0.659,0.659,0.659 ; 0.8)) \end{aligned}$ | 0.659 | (0.659,0.659,0.659) | $\begin{aligned} & ((0.659,0.659,0.659 ; 1), \\ & (0.659,0.659,0.659 ; 0.8)) \end{aligned}$ |
|  | C4 | 0.592 | (0.563,0.592,0.619) | $\begin{gathered} ((0.563,0.592,0.619 ; 1), \\ (0.578,0.592,0.606 ; 0.8)) \end{gathered}$ | 0.592 | (0.563,0.592,0.619) | $\begin{aligned} & ((0.563,0.592,0.619 ; 1), \\ & (0.578,0.592,0.606 ; 0.8)) \end{aligned}$ |
|  | C5 | 0.729 | (0.675,0.729,0.772) | $\begin{aligned} & ((0.675,0.729,0.772 ; 1), \\ & (0.702,0.729,0.752 ; 0.8)) \end{aligned}$ | 0.729 | (0.675,0.729,0.772) | $\begin{gathered} ((0.675,0.729,0.772 ; 1), \\ (0.702,0.729,0.752 ; 0.8)) \end{gathered}$ |
|  | C6 | 0.659 | (0.659,0.659,0.659) | $\begin{gathered} ((0.659,0.659,0.659 ; 1), \\ (0.659,0.659,0.659 ; 0.8)) \end{gathered}$ | 0.659 | (0.659,0.659,0.659) | $\begin{aligned} & ((0.659,0.659,0.659 ; 1), \\ & (0.659,0.659,0.659 ; 0.8)) \end{aligned}$ |
|  | C7 | 0.902 | (0.815,0.902,0.949) | $\begin{gathered} ((0.815,0.902,0.949 ; 1), \\ (0.854,0.902,0.934 ; 0.8)) \end{gathered}$ | 0.902 | (0.815,0.902,0.949) | $\begin{gathered} ((0.815,0.902,0.949 ; 1), \\ (0.854,0.902,0.934 ; 0.8)) \end{gathered}$ |
| 3.0 | C1 | 0.992 | (0.96,0.992,0.998) | $\begin{gathered} ((0.96,0.992,0.998 ; 1), \\ (0.971,0.992,0.998 ; 0.8)) \end{gathered}$ | 0.992 | (0.96,0.992,0.998) | $\begin{gathered} ((0.96,0.992,0.998 ; 1), \\ (0.971,0.992,0.998 ; 0.8)) \end{gathered}$ |
|  | C2 | 0.997 | (0.982,0.997,0.999) | $\begin{aligned} & ((0.982,0.997,0.999 ; 1), \\ & (0.989,0.997,0.999 ; 0.8)) \end{aligned}$ | 0.997 | (0.982,0.997,0.999) | $\begin{gathered} ((0.982,0.997,0.999 ; 1), \\ (0.989,0.997,0.999 ; 0.8)) \end{gathered}$ |
|  | C3 | 0.944 | (0.944,0.944,0.944) | $\begin{aligned} & ((0.944,0.944,0.944 ; 1), \\ & (0.944,0.944,0.944 ; 0.8)) \end{aligned}$ | 0.944 | (0.944,0.944,0.944) | $\begin{aligned} & ((0.944,0.944,0.944 ; 1), \\ & (0.944,0.944,0.944 ; 0.8)) \end{aligned}$ |
|  | C4 | 0.837 | (0.736,0.837,0.892) | $\begin{aligned} & ((0.736,0.837,0.892 ; 1), \\ & (0.795,0.837,0.868 ; 0.8)) \end{aligned}$ | 0.837 | (0.736,0.837,0.892) | $\begin{gathered} ((0.736,0.837,0.892 ; 1), \\ (0.795,0.837,0.868 ; 0.8)) \end{gathered}$ |
|  | C5 | 0.972 | (0.945,0.972,0.986) | $\begin{aligned} & ((0.945,0.972,0.986 ; 1), \\ & (0.961,0.972,0.98 ; 0.8)) \end{aligned}$ | 0.972 | (0.945,0.972,0.986) | $\begin{aligned} & ((0.945,0.972,0.986 ; 1), \\ & (0.961,0.972,0.98 ; 0.8)) \end{aligned}$ |
|  | C6 | 0.944 | (0.944,0.944, 0.944$)$ | $\begin{aligned} & (0.944,0.944,0.944 ; 1), \\ & (0.944,0.944,0.944 ; 0.8)) \end{aligned}$ | 0.944 | (0.944,0.944,0.944) | $\begin{gathered} ((0.944,0.944,0.944 ; 1), \\ (0.944,0.944,0.944 ; 0.8)) \end{gathered}$ |
|  | C7 | 1.000 | (0.997,1,1) | ((0.997,1,1;1),(0.999,1,1;0.8)) | 1.000 | (0.997,1,1) | ((0.997,1,1;1),(0.999,1,1;0.8)) |
| 5.0 | C1 | 1.000 | (0.996,1,1) | ((0.996,1,1;1),(0.997,1,1;0.8)) | 1.000 | (0.996,1,1) | ((0.996,1,1;1),(0.997,1,1;0.8)) |
|  | C2 | 1.000 | (0.999,1,1) | ((0.999,1,1;1),(0.999,1,1;0.8)) | 1.000 | (0.999,1,1) | ((0.999,1,1;1),(0.999,1,1;0.8)) |
|  | C3 | 0.993 | (0.993,0.993,0.993) | $\begin{aligned} & ((0.993,0.993,0.993 ; 1), \\ & (0.993,0.993,0.993 ; 0.8)) \end{aligned}$ | 0.993 | (0.993,0.993,0.993) | $\begin{aligned} & ((0.993,0.993,0.993 ; 1), \\ & (0.993,0.993,0.993 ; 0.8)) \end{aligned}$ |
|  | C4 | 0.966 | (0.936,0.966,0.98) | $\begin{gathered} ((0.936,0.966,0.98 ; 1), \\ (0.953,0.966,0.974 ; 0.8)) \end{gathered}$ | 0.966 | $(0.936,0.966,0.98)$ | $\begin{gathered} ((0.936,0.966,0.98 ; 1), \\ (0.953,0.966,0.974 ; 0.8)) \end{gathered}$ |
|  | C5 | 0.997 | (0.991,0.997,0.999) | $\begin{aligned} & ((0.991,0.997,0.999 ; 1), \\ & (0.995,0.997,0.998 ; 0.8)) \end{aligned}$ | 0.997 | (0.991,0.997,0.999) | $\begin{aligned} & ((0.991,0.997,0.999 ; 1), \\ & (0.995,0.997,0.998 ; 0.8)) \end{aligned}$ |
|  | C6 | 0.993 | (0.993,0.993,0.993) | $\begin{aligned} & ((0.993,0.993,0.993 ; 1), \\ & (0.993,0.993,0.993 ; 0.8)) \end{aligned}$ | 0.993 | (0.993,0.993,0.993) | $\begin{aligned} & ((0.993,0.993,0.993 ; 1), \\ & (0.993,0.993,0.993 ; 0.8)) \end{aligned}$ |
|  | C7 | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) | 1.000 | $(1,1,1)$ | ((1,1,1;1),(1,1,1;0.8)) |

