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The Effect of Bichromatic Potential on Thirring Instantons

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Anahtar Kelimeler
Soliton,
Instanton,
Bichromatic,
Nonlinear
dynamic.

Özet: Thirring model is a 2D conformally invariant pure fermionic model with no mass term. The model admits particle-like solutions and these solutions are instantonic in character. In this study, the effect of bichromatic potential on Thirring instantons is investigated to get a better idea of their nonlinear dynamical nature. For this purpose, the phase portraits are constructed. Under some parameter conditions, chaos is observed.

Bikromatik Potansiyelin Thirring İnstantonları Üzerine Etkisi

Keywords
Soliton,
İnstanton,
Bikromatik,
Lineer olmayan
dinamik.

Abstract: Thirring model iki boyutlu konformal invaryant kütlesiz saf bir fermiyonik modeldir. Model parçacık benzeri çözümler sergiler ve bu çözümler instanton karakterindedir. Bu çalışmada Thirring instantonlarının lineer olmayan dinamiğini daha iyi kavrayabilmek için bikromatik potansiyelin model üzerindeki etkisi incelenmiştir. Bu amaçla faz portreleri inşa edilmiş ve bazı sistem parametre değerleri için kaos gözlemlenmiştir.

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1. Introduction

It is known that solitons play an important role in many areas, ranging from theoretical physics to applied mathematics. They were discovered as undissipated surface waves on water and realized to obey nonlinear wave equations [1-2]. Instanton, one of the leading soliton types, has a finite action with zero energy. Instantons are classical topological solutions of the field equations of any given model in quantum field theories which lie behind the standard model of particles. [2]. Such solutions were obtained in exact analytical form for a variety of models including Quantum Chromo Dynamics (QCD). They have been considered as configurations of quantum fields that provide a tunnelling effect between the vacuums that have different topologies in space-time [3-5]. The name indicates that the fluctuations are confined to an instant in space-time [6]. They originate from topological structure of vacuum in non-abelian gauge field theories [6-7]. Despite their undoubted importance for the theory of strong and electroweak interactions, direct experimental evidence for instanton-induced processes have being lacking until now. It is hoped that an attentive analysis of Large Hadron Collider (LHC) data in CERN might bring experimental confirmation of such processes [8-11].

Thirring model is a well-known system of single self-interacting massless fermions in (1 + 1) space-time dimensions with the nonlinear self-interaction term [12]. The model is pure fermionic with a nonlinear coupling and it has conformal symmetry. It contains many of the typical features of the quantization of relativistic quantum field theories and plays a very fruitful role in the progress of field theory [12]. A class of spinor-type instanton solutions of conformally invariant pure spinor Thirring field equation was found by the spontaneous symmetry breaking of the conformal invariance [12,13]. We call these solutions as Thirring instantons. Recently, the role of the coupling constant in the evolution of the two dimensional spinor-type Thirring instantons has been investigated in phase space [14, 15]. The vector analysis and orientations of Thirring instantons have been studied [16]. Also there have been a number of studies on instantons (See some of these in Refs. [17-21]).

In this paper, the regular and chaotic behaviours of the spinor-type Thirring instantons are studied under the bichromatic potential to get more information about the quantum dynamics of spinor type instantons in vacuum. We investigate the effects of the external potential in the long time behaviours of the spinor-type Thirring instantons in phase space. To this end, we consider the general dynamics of Thirring nonlinear differential equations system formed by the use of Heisenberg ansatz with the addition of a bichromatical potential.

2. Model

The Thirring wave equation is

$$i\sigma_{\mu}\partial_{\mu}\psi + g\left(\bar{\psi}\psi\right)\psi = 0 \tag{1}$$

Where g is the positive coupling constant, σ_{μ} are Pauli matrices $(\mu = 1,2,3)$ and the fermion field $\psi(x)$ has scale dimension $\frac{1}{2}$ [22].

The complex form of the Euclidian configuration of Heisenberg ansatz [23] is

$$\psi = \left[ix_{\mu}\sigma_{\mu}\chi(s) + \varphi(s)\right]C \tag{2}$$

with an arbitrary spinor constant C. $\chi(s)$ and $\varphi(s)$ are real functions of $s = x^2 + t^2 \left(x_1 \equiv x, x_2 \equiv t\right)$. Inserting Eq. (2) into Eq. (1), with

$$i\sigma_{\mu}\partial_{\mu}\psi = \begin{bmatrix} -2\chi(s) - 2s\dfrac{\mathrm{d}\chi(s)}{\mathrm{d}s} \\ +2ix_{\mu}\sigma_{\mu}\dfrac{\mathrm{d}\varphi(s)}{\mathrm{d}s} \end{bmatrix} \bar{C}C \tag{3}$$

and

$$g(\bar{\psi}\psi)\psi = \begin{bmatrix} igx_{\mu}\sigma_{\mu}\chi(s) \\ \left(s\chi^2(s) + \varphi^2(s)\right) \\ +g\varphi(s) \\ \left(s\chi^2(s) + \varphi^2(s)\right)^{\frac{1}{3}} \end{bmatrix} (\bar{C}C)C, \tag{4}$$

where

$$(\bar{\psi}\psi) = \left(s\chi^2(s) + \varphi^2(s)\right)(\bar{C}C), \tag{5}$$

one obtains the following nonlinear differential equations system

$$2\chi(s) + 2s\frac{\mathrm{d}\chi(s)}{\mathrm{d}s} - g\varphi(s)\left[s\chi(s)^2 + \varphi(s)^2\right]\bar{C}C = 0 \tag{6a}$$

$$
\begin{aligned}
&2\frac{\mathrm{d}\varphi(s)}{\mathrm{d}s}+ \\
&g\chi(s)\left[s\chi(s)^2+\varphi(s)^2\right]^{1/3} \\
&\bar{C}C=0.
\end{aligned}
\tag{6b}
$$

Substituting $\chi = As^{-\sigma}F(u)$ and $\varphi = Bs^{-\tau}G(u)$, with $u=\ln(s)$ and $\sigma=\tau+\frac{1}{2}, \tau=\frac{1}{4}$ and $A^2=B^2$ [12].

$$
\begin{aligned}
&2A|s|^{-\sigma}F(u)-2s\sigma A|s|^{-\sigma-1}F(u)+ \\
&2sA|s|^{-\sigma}F(u)\frac{1}{s}- \\
&gB|s|^{-\tau}G(u)\begin{pmatrix} sA^2|s|^{-2\sigma}F(u)^2 \\ +B^2|s|^{-2\tau}G(u)^2 \end{pmatrix} \\
&\left(\bar{C}C\right)=0
\end{aligned}
\tag{7a}
$$

$$
\begin{aligned}
&-2\tau B|s|^{-\tau-1}G(u)+2B|s|^{-\tau}G(u)\frac{1}{s}+ \\
&gA|s|^{-\sigma}F(u)\begin{pmatrix} sA^2|s|^{-2\sigma}F(u)^2 \\ +B^2|s|^{-2\tau}G(u)^2 \end{pmatrix} \\
&\left(\bar{C}C\right)=0,
\end{aligned}
\tag{7b}
$$

we achieve the dimensionless form of the non-linear ordinary coupled differential equations system (7) as

$$
\begin{aligned}
&2\frac{dF(u)}{du}+\frac{1}{2}F(u) \\
&-\alpha AB\left(F(u)^2+G(u)^2\right)G(u)=0
\end{aligned}
\tag{8a}
$$

$$
\begin{aligned}
&2\frac{dG(u)}{du}-\frac{1}{2}G(u) \\
&-\alpha AB\left(F(u)^2+G(u)^2\right)F(u)=0
\end{aligned}
\tag{8b}
$$

Here F and G are dimensionless functions of u, α, A and B are constant.
The bichromatic potential is

$$
\begin{aligned}
V &= C_1\,\mathrm{Cos}^2\left(\omega_1 u\right) \\
&+C_2\,\mathrm{Cos}^2\left(\omega_2 u\right)
\end{aligned}
\tag{9}
$$

Especially, this potential is one of the potentials used to trap bosons in Bose-Einstein condensate. The bichromatic potential has two different frequencies. This property is suitable for double-well conditions of instantons. Also more parameters are used in numerical process. If we redefine Thirring Nonlinear Differential Equations System with the bichromatic potential to get more information about the quantum dynamics of spinor type instantons as

$$
\begin{aligned}
&2\frac{dF(u)}{du}+\frac{1}{2}F(u) \\
&-\alpha AB\left(F(u)^2+G(u)^2\right)G(u)=0
\end{aligned}
\tag{10a}
$$

$$
\begin{aligned}
&2\frac{dG(u)}{du}-\frac{1}{2}G(u) \\
&-\alpha AB\left(F(u)^2+G(u)^2\right)F(u) \\
&-C_1\,\mathrm{Cos}^2\left(\omega_1 u\right) \\
&+C_2\,\mathrm{Cos}^2\left(\omega_2 u\right) \\
&=0
\end{aligned}
\tag{10b}
$$

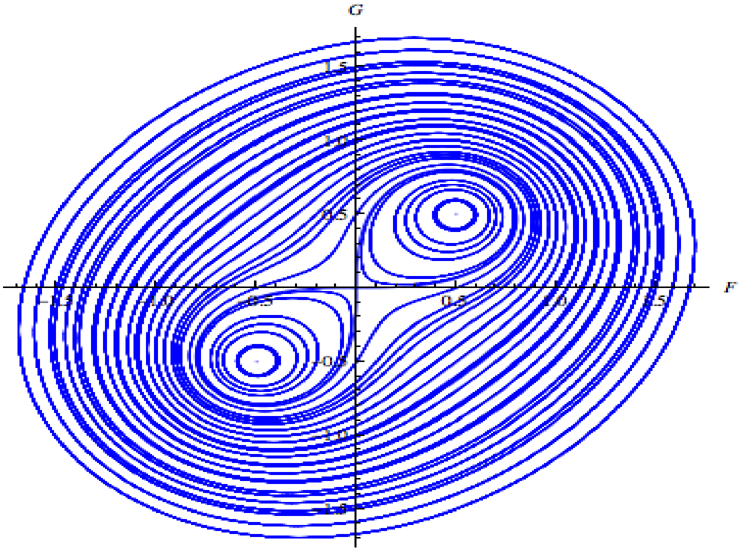
$$
\frac{dH(u)}{du}=\Omega
\tag{10c}
$$

With a H(u) function of u adding an extra dimension. It is known that , we need at least 3 dimensions to find chaos in problems involving continuous change described by differential equations [24]. Here Ω is a constant, C_1 and C_2 are the amplitudes of external potential and ω_1 and ω_2 are its frequencies respectively.

3. Numerical Results

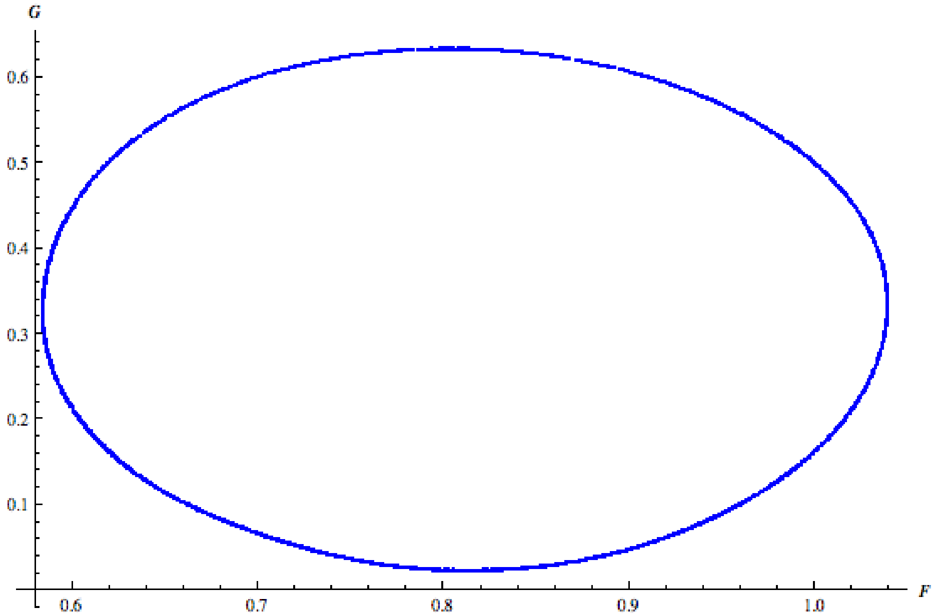
Methods from the viewpoint of nonlinear dynamics and chaos theory are quite useful in solving problems where chaos is present. It is difficult to obtain exact solutions directly for the Thirring model with potential. So we present some numerical results to get more information about the quantum dynamics of spinor type instantons in vacuum. We set $\alpha AB = 1$ in the paper because of the existence of the spinor-type Thirring instantons for this value [13]. Also the fixed points of sytem

$\left(-\frac{1}{2},-\frac{1}{2}\right)$ and $\left(\frac{1}{2},\frac{1}{2}\right)$ are used as initial conditions. TISEAN and Maxima's programming language are benefited in the paper.

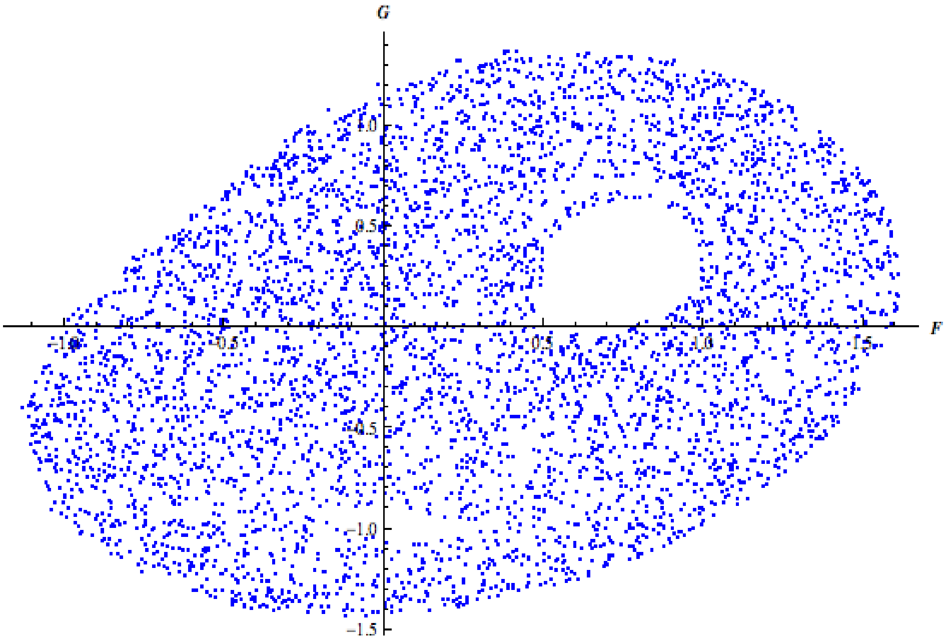


Şekil 1. Undamped Duffing type stability characterization of Thirring instantons for $\alpha AB = 1$

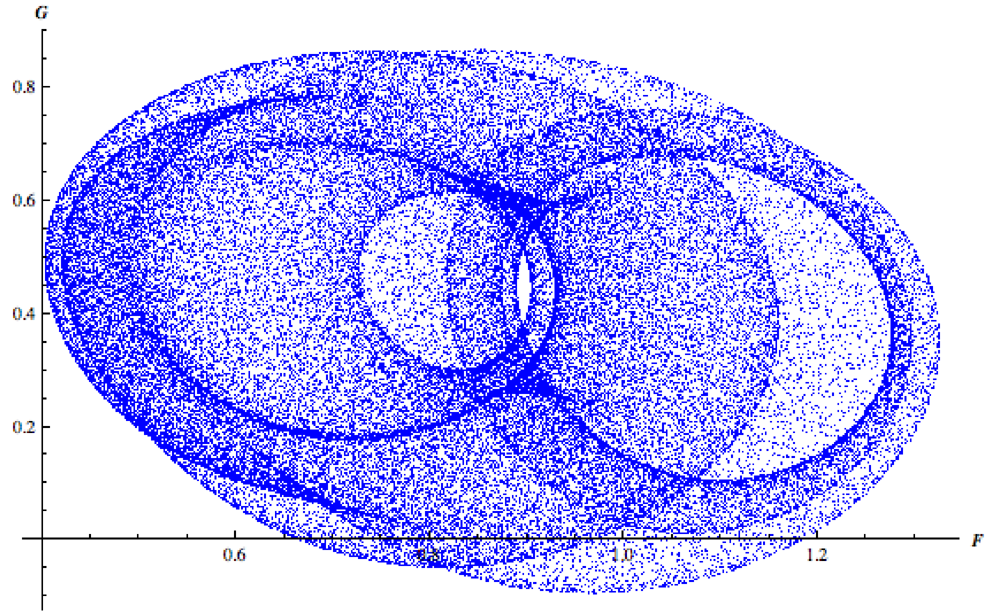
In Fig. 1, the phase space display of Thirring instantons is given for without potential. As can be seen from Fig.1, the phase space dynamics of 2D spinor type Thirring instantons possess a Duffing oscillator type steady-characterization without forcing and damping [14, 15]. It is well known that Duffing oscillator is a famous example describing the motion of a classical particle in a double well potential [25]. In Fig. 2, the phase space displays for different amplitude values are seen. For the weak potential in Fig. 2(a) the system shows regular behaviour. As we reinforce the amplitude, Fig. 2(b) shows a regular island embedded in the chaotic sea. It is interesting that the obtained phase space plot has a typical KAM-stability island. Namely, some originally periodic solutions remain regular while others start to behave chaotically [26]. Due to more amplified potential, Fig. 2(c) exhibit more chaotic regions. Also similar results are obtained for different frequency values as can be seen from Fig. 3. The system exhibits KAM-like behaviour in Fig. 3(b) and more chaotic regions in Fig. 3(c). So we can conclude that external potential having certain frequencies may change the undamped Duffing-type stability characteristics of spinor-type Thirring instantons in phase space for the same initial conditions. Thirring instantons can not maintain its stability for the above initial conditions, when the amplitude of external potential increase enough.



(a)

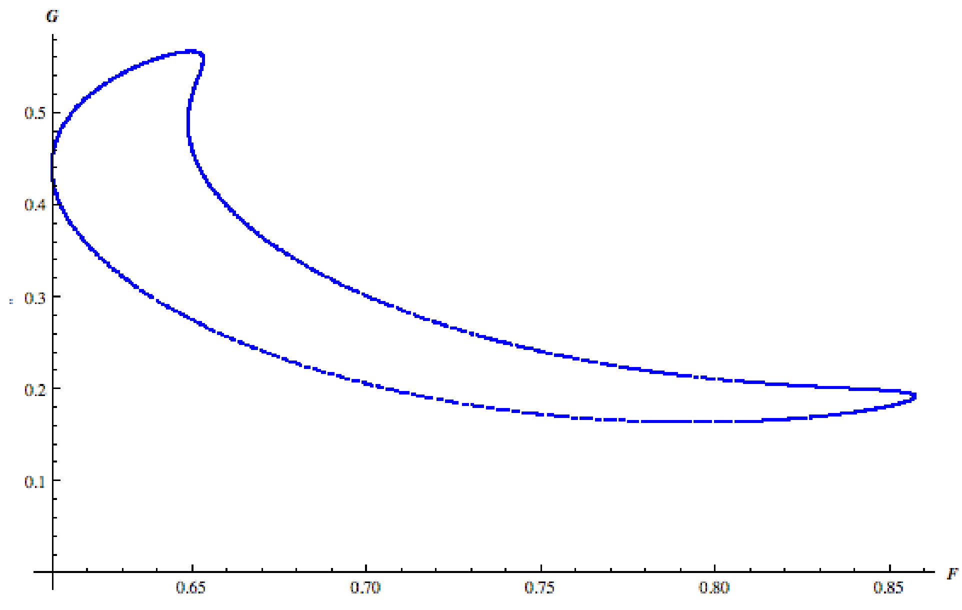


(b)

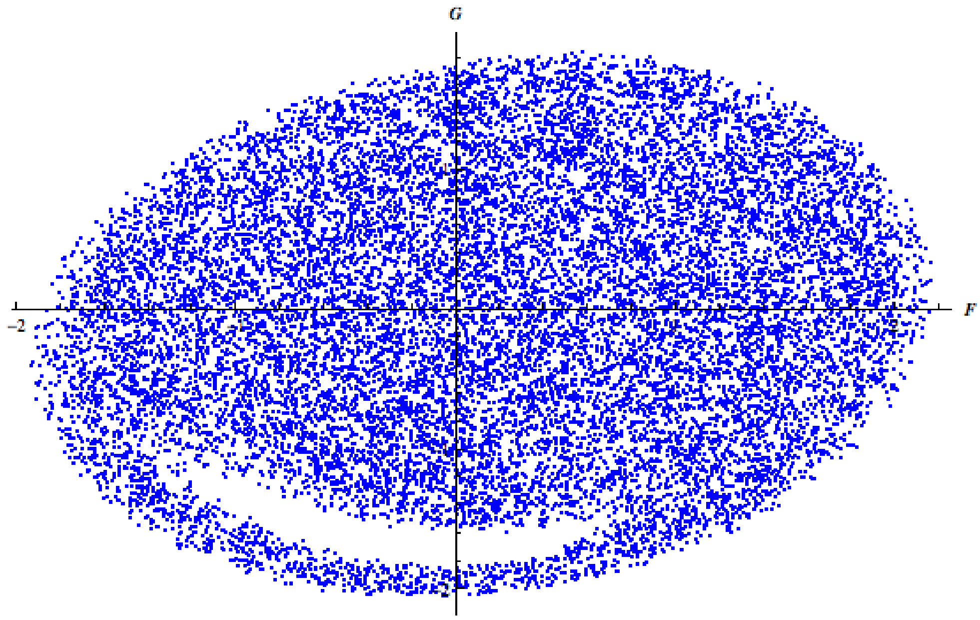


(c)

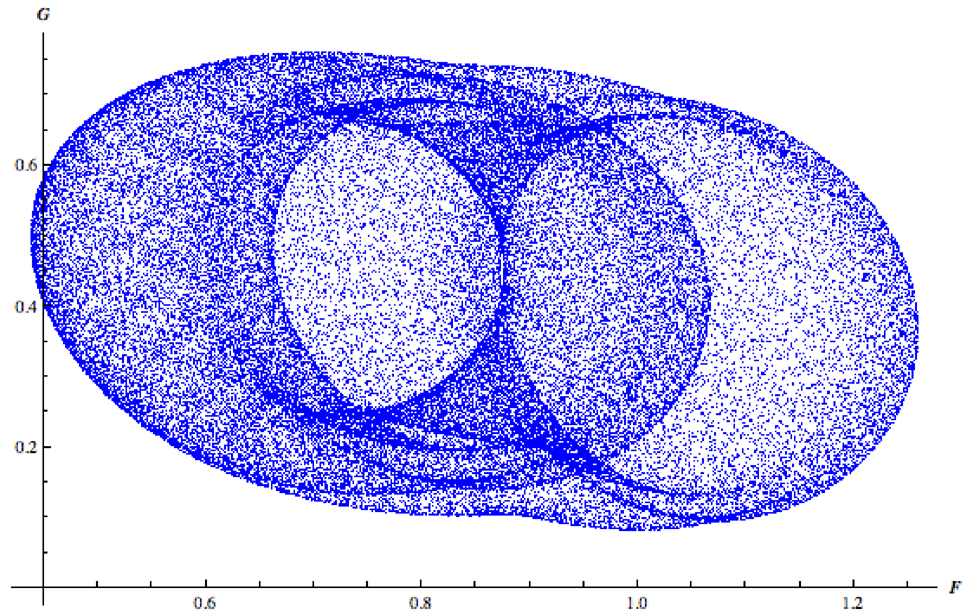
Şekil 2. Transition to chaos under the bichromatic potential for $\omega_1 = 0.2$ and $\omega_2 = 0.1$ (a) C_1=0.006, C_2=0.007 (b) C_1=0.1, C_2=0.09, (c) C_1=0.171, C_2=0.13



(a)



(b)



(c)

Şekil 3. Transition to chaos under the bichromatic potential for $\omega_1 = 0.1$ and $\omega_2 = 0.05$ (a) $C_1=0.006$, $C_2=0.007$ (b) $C_1=0.1$, $C_2=0.09$, (c) $C_1=0.171$, $C_2=0.13$

3. Discussion and Conclusion

It is known that Thirring model contains many of the typical features of the quantization of relativistic quantum field theories. Recently, many studies have been done on this model to understand its dynamical nature [14-16 and 27]. In this paper, we consider the Thirring nonlinear differential equations system formed by the use of Heisenberg ansatz and we investigate it under the bicromatical potential to understand how the behaviours of spinor type Thirring instantons could be affected. In the view of obtained results, we can conclude that the undamped Duffing-type stability characteristics of spinor-type Thirring instantons disappear depending on the external potential parameter values. The bichromatic potential destroys the regularity of system and generates a stochastic layer. Thirring instantons lies on this layer which correspond to irregular curves with potential, i.e., a number of chaotic instantons appears depending on the external potential parameter values [28-29].

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