Alan Turing (1912 - 1954) was born 92 years ago today. He says “We can only see a short distance ahead, but we can see plenty there that needs to be done” on his paper on the Turing test.

Solve only 5 of 6 questions below. YOU must write GOOD Mathematics

1. Construct a truth table for each of these compound statements
   (a) \((p \lor q) \rightarrow (p \land \neg r)\)
   (b) \(((p \lor q) \land \neg p) \rightarrow q\)

   Answer:

2. Find and explain the order \(\Theta\) (complexity in terms of number of comparisons) to sort a list of \(n\) elements using
   (a) Bubble sort
   (b) Insertion sort.
3. Prove that if \( n \) is an integer and \( n^3 + 5 \) is odd, then \( n \) is even using

(a) an indirect proof.

(b) a proof by contradiction.

Answer:

4. (a) Using the Euclidean Algorithm, find \( \text{gcd}(101, 1001) \), the greatest common divisor of 101 and 1001.

(b) From the algorithm, find the integers \( a, b \) so that \( \text{gcd}(101, 1001) = 101a + 1001b \).

Answer:
5. In how many ways can 100 candies be distributed among 5 children such that each child has at least 2 candies?

Answer:

6. Find an explicit formula for the Lucas numbers $L_n$ by solving the recurrence relation

$$L_n = L_{n-1} + L_{n-2}, \quad L_0 = 2, L_1 = 1.$$