1. (a) If \( f(x) \) has a multiple root at \( r \) with multiplicity three i.e. 
\[
f(x) = (x - r)^3 g(x), \quad \text{and} \quad g(r) \neq 0,
\]
show that Newton’s method is NOT a second order process for finding \( r \).

(b) Find a value of \( \lambda \) so that the following modified Newton’s method is at least second order process,
\[
x_{n+1} = x_n - \lambda \frac{f(x_n)}{f'(x_n)}.
\]
2. By using Gaussian elimination, find an equation of the circle through the points (1, 5), (−2, 3) and (2, −1). Show all your steps clearly.

3. Consider the nonlinear system

\[
\begin{align*}
x^2 - 2x - y + \frac{1}{2} &= 0 \\
x^2 + 4y^2 - 4 &= 0.
\end{align*}
\]

Use Newton’s method with the starting value \((x_0, y_0) = (2, 1/4)\) and compute \((x_1, y_1)\).
4. Find \( LU \) with \( L_{ii} = 1 \) and \( LDU \) with \( U_{ii} = 1 \), \( D \) diagonal matrix, factorization of \( H_3 \) where \( n \times n \) Hilbert matrix \( H_n \) has elements

\[
H_n = \frac{1}{i + j - 1}, \quad i, j = 1, 2, \ldots, n.
\]