• YOU must write GOOD Mathematics, clearly explaining each step of your proof. Otherwise, no objection!

1. Let $x \in \mathbb{R}^+$ and assume $x = q \times 2^m$ where $1 \leq q < 2$ in binary form and $-126 \leq m \leq 127$. If $x$ is represented (or approximated) in a 32-bit computer by $x_-$, find the bound for the relative error.

2. Set $x = \frac{1}{3}$. Find the machine numbers $x_-$ and $x_+$ as in the above form. So, which one is $\text{fl}(x)$?

3. In the Bisection method, prove that $c_n$, the midpoints of the intervals where the function changes sign converges to the root $r$ with a rate of convergence $O(2^{-n})$. That is

$$c_n = r + O(2^{-n}).$$