MATH 205 ANALYTIC GEOMETRY	Answers for Quiz 1	27.10.2003
----------------------------	--------------------	------------

1. Given the vertices of a quadrangle ABCD, A = (1,1), B = (4,1), C = (3,-2) and D = (0,-2), determine whether they form a rectangle, a rhombus or a parallelogram.

SOLUTION Let us find the distances;

$$|AB| = 3$$
,  $|BC| = \sqrt{(4-3)^2 + (1+2)^2} = \sqrt{10}$ ,  $|CD| = 3$ ,  $|DA| = \sqrt{10}$ 

The lines between the points A, B and the points C, D are parallel. Further, the line between the points A, D is not vertical. Thus, from the evidences above the quadrangle is a parallelogram.

2. Show that a triangle with vertices  $(x_1, y_1), (x_2, y_2)$  and  $(x_3, y_3)$  has area

$$\frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_2y_1 - x_3y_2| = \begin{vmatrix} 1 \\ 2 \\ x_1 \\ x_2 \\ x_3 \\ y_3 \\ 1 \end{vmatrix} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 \\ x_3 \\ y_3 \\ 1 \end{vmatrix}$$

Hint: Name the vertices counterclockwise and draw the min-max box of the triangle. That is a rectangle whose sides are parallel to the axes and pass through the vertices.

Solution



The area of the triangle R is the differences of the area of min-max box and the sum of areas of the other three triangles. So, we may write

$$R = (x_2 - x_1)(y_3 - y_1) - \frac{1}{2}(x_2 - x_1)(y_2 - y_1) - \frac{1}{2}(x_2 - x_3)(y_3 - y_2) - \frac{1}{2}(x_3 - x_1)(y_3 - y_1)$$

Expanding and then cancelling the terms in the latter equation gives the desired result.