MATH 205 ANALYTIC GEOMETRY   **Quiz 1**   17.10.2005

Name   **Student No.**   **Sign**

You will not get any points if your answer is wrong, that is no points to your explanations if your answer is wrong. And of course no points to a correct answer if your explanation or proof is not correct or clear.
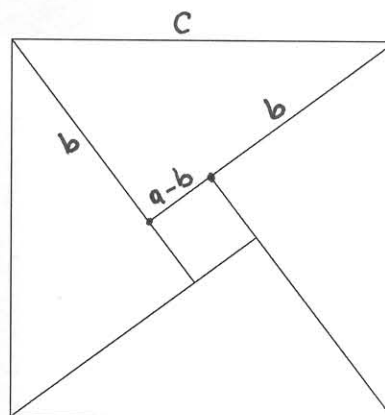
YOU must write GOOD Mathematics

1. Below is square of side length c partitioned with a square of side length $a - b$ in the center. Prove the Pythagorean Theorem using the figure.

Writing the area of square in two different ways gives

$$c^2 = (a-b)^2 + 4 \cdot \frac{a.b}{2} \quad \text{since}$$

thee are four identical right triangles. Then

$$c^2 = a^2 - 2ab + b^2 + 2ab = a^2 + b^2.$$

2. Let $(r^2 - s^2, 2rs, r^2 + s^2)$, $r \geqslant s > 0$ be a Pythagorean triple. Find its corresponding rational point on the unit circle.
Solution:

Since $(r^2 - s^2)^2 + (2rs)^2 = (r^2 + s^2)^2$,

$\left(\dfrac{r^2 - s^2}{r^2 + s^2}, \dfrac{2rs}{r^2 + s^2} \right)$ is the required rational point on the unit circle.

3. *Use parametric equation of a line* to find the midpoint of the points $A = (x_0, y_0)$, $B = (x_1, y_1)$.
Solution:

We showed that a point (x,y) on a line is shown by

$$x = x_0 + t(x_1 - x_0), \quad y = y_0 + t(y_1 - y_0)$$

where $A = (x_0, y_0)$, $B = (x_1, y_1)$ are on the line. Then taking $t = 1/2$ gives the midpoint of A, B i.e.

$$x = x_0 + \frac{1}{2}(x_1 - x_0) = \frac{x_0 + x_1}{2}.$$

$$y = y_0 + \frac{1}{2}(y_1 - y_0) = \frac{y_0 + y_1}{2}.$$