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MATH 205 ANALYTIC GEOMETRY **Answers for Quiz 2** 03.11.2003

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1. Find equations (general form $ax+by+c=0$) of the three sides of the triangle with vertices $A=(-1,-2), B=(3,0)$ and $C=(1,4)$.

SOLUTION The general form of a line passing thorough the points $(x_1,y_1),(x_2,y_2)$ is

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}.$$

Thus the line L_1 between $(-1,-2)$ and $(3,0)$ is

$$L_1;\quad \frac{y+2}{x+1}=\frac{0+2}{3+1}\quad\Longleftrightarrow\quad x-2y-3=0.$$

The line L_2 between $(3,0)$ and $(1,4)$ is

$$L_2;\quad \frac{y-0}{x-3}=\frac{4-0}{1-3}\quad\Longleftrightarrow\quad -2x-y+6=0.$$

The line L_3 between $(1,4)$ and $(-1,-2)$ is

$$L_3;\quad \frac{y-4}{x-1}=\frac{-2-4}{-1-1}\quad\Longleftrightarrow\quad 3x-y+1=0.$$

2. Find equations (depending on a parameter t) of the medians and their intersection (*the centroid*) of the triangle in the previous question.

SOLUTION Let M_1,M_2,M_3 be the midpoints of the points A,B and B,C, and C,A respectively. Then, clearly

$$M_1=(1,-1),\quad M_2=(2,2),\quad M_3=(0,1).$$

The line segment $l_1(x,y)$ from C to M_1 is

$$l_1(x,y);\quad (1+t(1-1),4+t(-1-4))=(1,4-5t),\quad 0\leqslant t\leqslant 1.$$

The line segment $l_2(x,y)$ from A to M_2 is

$$l_2(x,y);\quad (-1+t(2-(-1)),-2+t(2-(-2)))=(-1+3t,-2+4t),\quad 0\leqslant t\leqslant 1.$$

The line segment $l_3(x,y)$ from B to M_3 is

$$l_3(x,y);\quad (3+t(0-3),0+t(1-0))=(3-3t,t),\quad 0\leqslant t\leqslant 1.$$

The centroid (barycenter) of the triangle can be found on any $l_1\cap l_2$, $l_2\cap l_3$ or $l_1\cap l_3$. Using the first one we obtain $1=-1+3t$ and hence the ratio is $t=2/3$. Now putting this ratio on any line gives the barycenter of the triangle

$$(1,4-5\frac{2}{3})=(1,\frac{2}{3}).$$