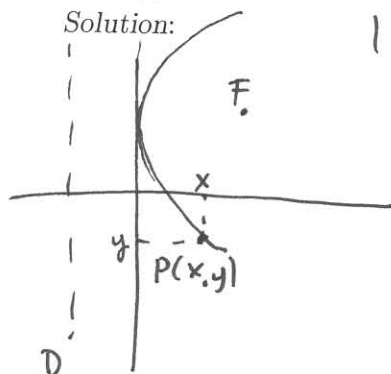


You will not get any points if your answer is wrong, that is no points to your explanations if your answer is wrong. And of course no points to a correct answer if your explanation or proof is not correct or clear.

YOU must write GOOD Mathematics

1. Use geometric definition to find the equation of the parabola with focus $F = (2, 2)$ and directrix $x = -2$.

Solution:



$|PF| = |PD|$ Let $P(x, y)$ be an arbitrary point on the parabola.

$$\text{Then } \sqrt{(x-2)^2 + (y-2)^2} = x+2.$$

Squaring both sides gives

$$x^2 - 4x + 4 + y^2 - 4y + 4 = x^2 + 4x + 4$$

which simplifies to

$$x = \frac{1}{8} (y^2 - 4y + 8)$$

2. Find the foci, vertices and center focus distance of the conic

$$x^2 + 4y^2 - 2x - 8y + 1 = 0.$$

Solution:

Completing to perfect squares gives

$$(x-1)^2 - 1 + 4(y-1)^2 - 4 + 1 = 0 \quad \text{or}$$

$$\frac{(x-1)^2}{4} + \frac{(y-1)^2}{1} = 1$$

an ellipse whose center is $(1, 1)$ has

semimajor axis $a = 2$

semiminor axis $b = 1$ and $c = \sqrt{2^2 - 1} = \sqrt{3}$ center focus distance.

Thus vertices are $(1 \pm 2, 1)$, foci $(1 \pm \sqrt{3}, 1)$.