MATH 205 ANALYTIC GEOMETRY **Answers for Quiz 3** 18.11.2003

Name **Student No.**

YOU must write GOOD Mathematics, clearly explaining each step of your proof. Otherwise, your objection may not be accepted.

Find the center, foci, vertices, asymptotes and radius, as appropriate, of the conic section and then sketch it.

1. $4x^2 + y^2 - 8x + 4y + 4 = 0$.

   SOLUTION Completing the above equation into a sum of perfect squares gives

   $$4(x-1)^2 + (y+2)^2 = 4 \quad \text{so that} \quad (x-1)^2 + \frac{(y+2)^2}{4} = 1.$$

   Let us move the xy coordinate system 1 unit right and 2 units down by setting $\overline{x} = x - 1$ and $\overline{y} = y + 2$ to have a new coordinate system $\overline{xy}$. Now, on $\overline{xy}$, the latter equation is an ellipse

   $$\overline{x}^2 + \frac{\overline{y}^2}{4} = 1$$

   whose: focal axes is vertical, has center $(0,0)$ and the center focus distance $a = 2$, vertices $(0, \mp 2)$, $b = 1$ so that $c = \sqrt{a^2 - b^2} = \sqrt{3}$ center to focus distance, foci $(0, \mp\sqrt{3})$. Moving the axes to prescribed location, where $x = \overline{x} + 1$ and $y = \overline{y} - 2$ gives the details on xy; *center* $(1, -2)$, *foci* $(1, \mp\sqrt{3} - 2)$, *vertices* $(1, \mp 2 - 2)$. To sketch the ellipse, first draw it in the system $\overline{xy}$ and then move it 1 unit right and 2 units down.

2. $x^2 - y^2 + 4x - 6y - 6 = 0$.

   SOLUTION Completing the above equation into a difference of perfect squares gives

   $$(x+2)^2 - (y+3)^2 = 1.$$

   Let us move the xy coordinate system 2 units left and 3 units down by setting $\overline{x} = x + 2$ and $\overline{y} = y + 3$ to have a new coordinate system $\overline{xy}$. Now, on $\overline{xy}$, the latter equation is a hyperbola

   $$\overline{x}^2 - \overline{y}^2 = 1$$

   whose: focal axes is horizontal, has center $(0,0)$, with vertices $(\mp 1, 0)$, $a = 1, b = 1$ and the focus center distance $c = \sqrt{a^2 + b^2} = \sqrt{2}$, foci $(\mp\sqrt{2}, 0)$ and asymptotes $\overline{y} = \mp\overline{x}$. Moving the axes to prescribed location, where $x = \overline{x} - 2$ and $y = \overline{y} - 3$ gives the details on xy; *center* $(-2, -3)$, *foci* $(\mp\sqrt{2} - 2, -3)$, *vertices* $(\mp 1 - 2, -3)$ and *asymptotes* $y - 3 = \mp(x - 2)$, i.e. $y = x + 1$ and $y = -x + 5$. To sketch the hyperbola, first draw it in the system $\overline{xy}$ and then move it 2 units left and 3 units down.