

You will not get any points if your answer is wrong, that is no points to your explanations if your answer is wrong. And of course no points to a correct answer if your explanation or proof is not correct or clear.

YOU must write GOOD Mathematics

1. Verify the focus-directrix formula $|PF| = e|PD|$ for $x^2 - y^2 = 1$.

Solution:

the $a=1, b=1$ and $e = \frac{\sqrt{2}}{1} = \frac{c}{a}$, directrices $x = \pm \frac{1}{\sqrt{2}} = \frac{a^2}{c}$
foci $(\pm\sqrt{2}, 0)$

Then $|PF| = e|PD|$ gives

$$\sqrt{(x-\sqrt{2})^2 + (y-0)^2} = \sqrt{2} \left(x - \frac{1}{\sqrt{2}}\right) \text{ or}$$

$$(x-\sqrt{2})^2 + (y-0)^2 = 2 \left(x - \frac{1}{\sqrt{2}}\right)^2$$

simplifies to $x^2 - y^2 = 1$, the hyperbola whose focal axis is horizontal.

2. Find eccentricity and directrix of

$$4x^2 + y^2 - 4x + 2y = 2.$$

Solution:

Completing the squares gives

$$4\left(x - \frac{1}{2}\right)^2 - 1 + (y+1)^2 - 1 = 2,$$

$$\left(x - \frac{1}{2}\right)^2 + \frac{(y+1)^2}{4} = 1 \text{ or ellipse whose focal axis}$$

is vertical!

Center $\left(\frac{1}{2}, -1\right)$, semi major axis $a=2$
semi minor axis $b=1$

$$\text{center focus distance } c = \sqrt{2^2 - 1} = \sqrt{3}$$

Hence eccentricity e is $e = \frac{c}{a} = \frac{\sqrt{3}}{2}$

Directrices are horizontal

$$y = -1 \pm \frac{4}{\sqrt{3}}.$$