# İzmir University of Economics, Department of Mathematics

MATH 205 ANALYTIC GEOMETRY **Answers for Quiz 4** 01.12.2003

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1. Find *center, foci, vertices, eccentricity, directrix* of the curve

$$9x^2 + 4(y-1)^2 = 36.$$

SOLUTION The conic may be written is the form

$$\frac{x^2}{4} + \frac{(y-1)^2}{9} = 1.$$

We translate the axis by putting $x' = x, y' = y - 1$ which means moving 1 unit up. Notice that any point in the plane is related by $(x, y) \longleftrightarrow (x', y' + 1)$. So, we now have

$$\frac{x'^2}{4} + \frac{y'^2}{9} = 1,$$

which is an ellipse with a *vertical* focal axis. In the $x'y'$ coordinates, the ellipse has; *center* $(0, 0)$, *vertices* $(0, \pm 3)$, *center-focus distance* $c = \sqrt{9-4} = \sqrt{5}$, foci $(0, \pm\sqrt{5})$, *eccentricity* $e = \sqrt{5}/3$, *directrix* $y' = \pm\frac{9}{\sqrt{5}}$. By translation of axis, the eccentricity and center-focus distance remain unchanged. In the $xy$ coordinates, *vertices* $(0, 1 \pm 3)$, foci $(0, 1 \pm \sqrt{5})$ and *directrix* $y = 1 \pm \frac{9}{\sqrt{5}}$

2. Find a new representation of $x^2 + 4xy - 2y^2 - 6 = 0$ after rotating through an angle $\alpha = \arctan(1/2)$. Sketch the curve, showing both the old and new coordinate systems.

SOLUTION We have the equations of rotation from $xy$ coordinates to $x'y'$ coordinates

$$x = x' \cos\alpha - y' \sin\alpha$$
$$y = x' \sin\alpha + y' \cos\alpha$$

Notice that the rotation is by an acute angle $\alpha$ whose tangent is $1/2$. By forming a right triangle whose vertical sides has length 1 and 2 and hence hypothenuse length $\sqrt{5}$, we find that

$$\sin\alpha = \frac{1}{\sqrt{5}} \quad \text{and} \quad \cos\alpha = \frac{2}{\sqrt{5}},$$

and it follows from the equations of rotation that

$$x = \frac{2x' - y'}{\sqrt{5}} \quad \text{and} \quad y = \frac{x' + 2y'}{\sqrt{5}}.$$

Replacing the latter equation in the original conic, we have

$$\frac{(2x'-y')^2}{5} + 4\frac{2x'-y'}{\sqrt{5}}\frac{x'+2y'}{\sqrt{5}} - 2\frac{(x'+2y')^2}{5} - 6 = 0,$$

and is simplified to give

$$2x'^2 - 3y'^2 = 6 \quad \text{or} \quad \frac{x'^2}{3} - \frac{y'^2}{2} = 1.$$

This is a hyperbola whose *focal axis is horizontal* and has properties ; *center* $(0, 0)$, *vertices* $(\pm\sqrt{3}, 0)$, *center-focus distance* $\sqrt{5}$, foci $(\pm\sqrt{5}, 0)$ and *asymptotes* $y' = \pm\sqrt{2/3}x'$, *eccentricity* $e = \sqrt{5/3}$, *directrix* $x' = \pm\frac{3}{\sqrt{5}}$ in the $x'y'$ coordinates.

Those who wonders the corresponding values in the $xy$ coordinate may use the rotation equations to find them.