İzmir University of Economics, Department of Mathematics

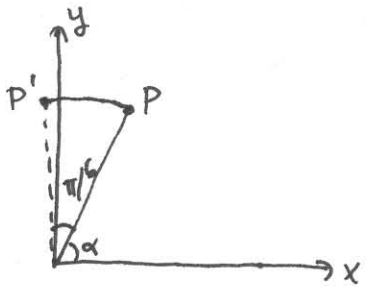
MATH 205 ANALYTIC GEOMETRY | Quiz 4 | 21.11.2005

Name | Student No. | Sign

You will not get any points if your answer is wrong, that is no points to your explanations if your answer is wrong. And of course no points to a correct answer if your explanation or proof is not correct or clear.

YOU must write GOOD Mathematics

1. Rotate the point $P(1,2)$ by $\pi/6$ using the *rotation matrix* to find the new location of P.
Solution:



rotation θ is $\pi/6$

P' has coordinates $(\bar{x}, \bar{y})$

$\bar{x} = r\cos(\alpha+\theta) = x\cos\theta - y\sin\theta$

$\bar{y} = r\sin(\alpha+\theta) = y\cos\theta + x\sin\theta$

since $x = r\cos\alpha$ and $y = r\sin\alpha$

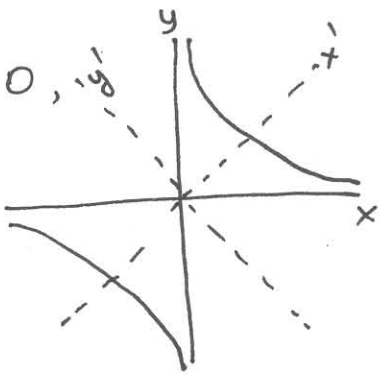
Then $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}\cos\frac{\pi}{6} & -\sin\frac{\pi}{6}\\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6}\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}$

$= \left(\frac{\sqrt{3}}{2}-1,\ \frac{1}{2}+\sqrt{3}\right)$, lies in the 2nd quadrant since $\frac{\sqrt{3}}{2}-1<0$.

2. Find the standard form of the conic $4xy = 1$ by a suitable *rotation of axis.*
Solution:

In the eqn $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ we have

$A=0,\ B=4,\ C=0,\ D=0,\ E=0,\ F=-1.$



Then rotation θ of axis has $\cot 2\theta = \frac{A-C}{B} = 0$,

$2\theta = \frac{\pi}{2}$, $\theta = \frac{\pi}{4}$ //. Then we write

$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\cos\frac{\pi}{4} & -\sin\frac{\pi}{4}\\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4}\end{pmatrix}\begin{pmatrix}x'\\ y'\end{pmatrix}$. Namely

$x = \frac{1}{\sqrt{2}}(x'-y')$ and $y = \frac{1}{\sqrt{2}}(x'+y')$. Thus we have

$4\cdot\frac{1}{\sqrt{2}}(x'-y')\cdot\frac{1}{\sqrt{2}}(x'+y') = 1$ which is

$2(x'^2-y'^2)=1$ or $\frac{x'^2}{1/2} - \frac{y'^2}{1/2} = 1$, a hyperbola whose center is $(0,0)$