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MATH 205 ANALYTIC GEOMETRY **Answers for Quiz 5** 15.12.2003

Name **Student No.**

1. Find a function or cartesian equation for the parametric curve

$$x = \tan t, \quad y = \sec t \qquad -\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{4}.$$

SOLUTION Since $\tan^2 t + 1 = \sec^2 t$, we obtain that $y^2 - x^2 = 1$ which is a hyperbola whose focal axis is vertical. At the initial point $t = -\pi/4$, we have $(x, y) = (-1, \sqrt{2})$ and at the end point $t = \pi/4$, we have $(x, y) = (1, \sqrt{2})$. It is the upper branch of the hyperbola traversed from left to right. So, the above parametric equation parametric has the cartesian form

$$y^2 - x^2 = 1, \qquad -1 \leqslant x \leqslant 1, \quad y \geqslant 0.$$

2. Find the parametric equation for the curve $9x^2 + 4y^2 = 36$, traversed in the *clockwise* direction.
SOLUTION First we have, $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Then, since $\cos^2 t + \sin^2 t = 1$, the parametrization $x = \mp 2\cos t \quad y = \mp 3\sin t \quad 0 \leqslant t \leqslant 2\pi$ will do the job. In order to decide which signs are to be taken for the sake clockwise direction on the ellipse, test a few easy points. We require $t = 0, \pi/2$ give the points $(2, 0), (0, -3)$. Now it is clear that their signs must be opposite and thus the parametrization of the ellipse above is

$$x = 2\cos t, \quad y = -3\sin t, \qquad 0 \leqslant t \leqslant 2\pi.$$

Note that the other choice also gives the correct answer but have a different initial points which does not matter.

3. Find the parametric equation for the curve $x^{2/3} + y^{2/3} = 1$, traversed from the point $(1, 0)$ to $(0, 1)$.
SOLUTION The fact that since $\cos^2 t + \sin^2 t = 1$ lead us to set $X = x^{1/3}, Y = y^{1/3}$. Then we have, $X^2 = x^{2/3}, Y^2 = y^{2/3}$. Now, setting $X = \cos t, Y = \sin t$ gives $X^2 + Y^2 = 1$. Thus, we obtain the parametrization $x = X^{2(3/2)} = \cos^3 t$ and $y = Y^{2(3/2)} = \sin^3 t$. The initial and the end point clearly give the domain $0 \leqslant t \leqslant \pi/2$. Therefore,

$$x = \cos^3 t, y = \sin^3 t, \qquad 0 \leqslant t \leqslant \pi/2$$

is the parametrization for the above curve.