Матн 205	Analytic Geometry	Answers for Quiz 6	22.12.2003
Name	Student No.		

1. The Witch of Agnesi curve is known by the parametric equation

$$x = at$$
, $y = \frac{a}{1+t^2}$, $t \in (-\infty, \infty)$, a is a fixed constant.

Find a point where the tangent line is horizontal.

SOLUTION We should have $\frac{dy}{dx} = 0$ at some t. The derivatives are

$$\frac{dy}{dt} = -2at(1+t^2)^{-3}$$
, and $\frac{dx}{dt} = a$,

and hence

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2t}{(1+t^2)^3} = 0 \quad \text{when} \quad t = 0.$$

Thus from the parametric equation, the point at t = 0 is (0, a).

2. The *cardioid* curve is given by

 $x = 2a\cos t - a\cos 2t$, $y = 2a\sin t - a\sin 2t$, $t \in [0, 2\pi]$, a is a fixed constant.

Find the parametric form of the tangent line at $t = \pi/2$. SOLUTION Recall that the parametric line equation at a point p is

$$x = x(p) + x'(p)t$$
 $y = y(p) + y'(p)t.$

At $p = \pi/2$ we the coordinates of

$$x = 2a\cos{\pi/2} - a\cos{\pi} = a, \quad y = 2a\sin{\pi/2} - a\sin{\pi} = 2a$$

The derivatives are

$$x'(t) = \frac{dx}{dt} = -2a\sin t + 2a\sin 2t \big|_{t=\pi/2} = -2a, \quad y'(t) = \frac{dy}{dt} = 2a\cos t - 2a\cos 2t \big|_{t=\pi/2} = 2a$$

Thus the parametric equation of a line passing thorough the point (a, 2a), having the above derivatives is

$$x = a - 2at \qquad y = 2a + 2at.$$

3. Find the slope of the curve at t = 1,

$$xt = \sqrt{5 - \sqrt{t}}, \qquad y(t-1) = \ln y$$

if x and y are implicitly differentiable functions of t.

SOLUTION Differentiating both sides with respect to t gives

$$\frac{d}{dt}(xt) = \frac{d}{dt}(\sqrt{5-\sqrt{t}}) \quad \text{and} \quad t\frac{dx}{dt} + x = \frac{1}{2\sqrt{5-\sqrt{t}}}\frac{-1}{2\sqrt{t}}$$

and at $t = 1 \implies x = 2$, dx/dt = -17/8. Differentiating both sides of the second implicit function with respect to t gives

$$\frac{d}{dt}(y(t-1)) = \frac{d}{dt}(\ln y) \quad \text{and} \quad t\frac{dy}{dt} + y - \frac{dy}{dt} = \frac{1}{y}\frac{dy}{dt}$$

and at $t = 1 \Longrightarrow y = 1$, dy/dt = 1. Thus dy/dx = -8/17.