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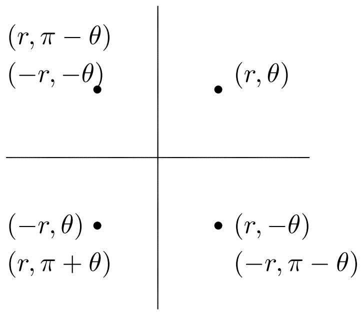
MATH 205 ANALYTIC GEOMETRY **Solutions for Quiz 7** 12.01.2004

Name **Student No.**

1. Find all the symmetries of the polar curve *lemniscate*

$$r^2 = 4\cos 2\theta.$$



Visit the famous curves index

`http://www-history.mcs.st-andrews.ac.uk/history/Curves/Curves.html`

SOLUTION Define the function $F(r,\theta) = r^2 - 4\cos 2\theta$. Since $\cos(-\theta) = \cos\theta$ we have $F(r,\theta) = F(r,-\theta)$ giving the symmetry with respect to x-axis. Since $(-r)^2 = r^2$, we have $F(r,\theta) = F(-r,\theta)$ giving the symmetry with respect to y-axis. These together implies the symmetry with respect to *origin*.

2. Find the cartesian form $F(x,y) = 0$ of the above curve *lemniscate*.
SOLUTION Since $r^2 = x^2 + y^2$ and $x = r\cos\theta$, $y = r\sin\theta$, and using the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$, we obtain that

$$r^2 = 4\cos 2\theta \Leftrightarrow r^2 = 4\cos^2\theta - 4\sin^2\theta \Leftrightarrow r^4 = 4r^2\cos^2\theta - 4r^2\sin^2\theta \Leftrightarrow (x^2+y^2)^2 = (2x)^2 - (2y)^2.$$

This gives the implicit form

$$x^4 + y^4 + 2x^2y^2 + 4y^2 - 4x^2 = 0.$$