MATH 205 ANALYTIC GEOMETRY **Quiz 7** 19.12.2005

Name **Student No.** **Sign**

You will not get any points if your answer is wrong, that is no points to your explanations if your answer is wrong. And of course no points to a correct answer if your explanation or proof is not correct or clear.

YOU must write GOOD Mathematics

1. Find the standard cartesian equation of the conic

$$r = \frac{4}{2 + \frac{1}{2}\cos\theta},$$

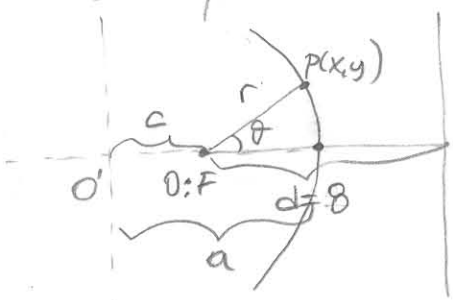
whose pole is at a focus.
Solution:

The polar equation for a conic is

$r = \dfrac{ed}{1+e\cos\theta}$ where d is focus directrix distance, eccentricity e.

Hence $r = \dfrac{4}{2(1+\frac{1}{4}\cos\theta)} = \dfrac{2}{1+\frac{1}{4}\cos\theta}$, $e=\frac{1}{4}$. Conic is ellipse

$\cos\theta$ implies conic has horizontal focal axis. We have



$e = \frac{c}{a} = \frac{1}{4}$ and $ed = 2$,

$d = 8$.

The cartesian coordinates whose origin is O' has $c+8 = \frac{a^2}{c}$; center - directrix distance

So $\frac{a^2-c^2}{c} = 8$ and $a = 4c$ gives $\frac{15c^2}{c} = 8$, $c = \frac{8}{15}$

Hence $a = \frac{32}{15}$. Since $b^2 = a^2 - c^2$, $b^2 = \left(\frac{32}{15}\right)^2 - \left(\frac{8}{15}\right)^2$,

$b = \frac{\sqrt{960}}{15}$

So, The cartesian eqn. for the ellipse is

$\dfrac{x'^2}{(\frac{32}{15})^2} + \dfrac{y'^2}{\frac{960}{15^2}} = 1$ and move c units, $\dfrac{(x-\frac{8}{15})^2}{(\frac{32}{15})^2} + \dfrac{y^2}{\frac{960}{15^2}} = 1.$