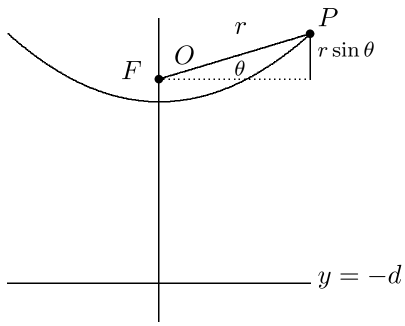
Math 205 Analytic Geometry **Solutions for Quiz 8** 22.01.2004

Name **Student No.**

1. Show that the conic with focus at the origin, directrix $y = \pm d$, and eccentricity e has polar equation
$$r = \frac{ed}{1 \pm e\sin\theta}.$$

Solution



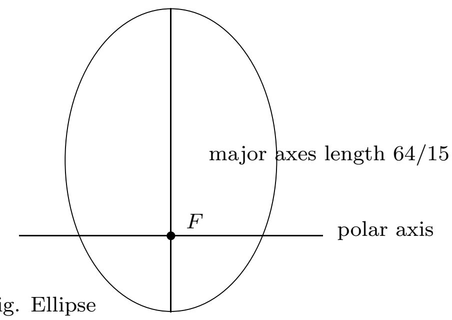


Fig. Ellipse

Place the origin (pole) at one focus of a conic section. We use focus-directrix formula $|PF| = e|PD|$. The distance between the point P and the directrix $y = -d$ is $|PD| = d + r\sin\theta$. Thus, we have
$$r = e(d + r\sin\theta) \quad \text{or} \quad r = ed + er\sin\theta \Longleftrightarrow r = \frac{ed}{1 - e\sin\theta}.$$
Notice that we measure the angle θ counterclockwise direction from the polar axis!

2. Graph the conic
$$r = \frac{8}{4 + \sin\theta},$$
placing the pole at a focus and the polar axis along the positive x-axis. What is the value of a in the cartesian form of the conic ?

Solution First write the equation as in the above form
$$r = \frac{2}{1 + \frac{1}{4}\sin\theta}.$$
This says, the curve is an ellipse with eccentricity $e = 1/4$ whose major axis is vertical. In order to find the the length a of semi major axis
$$r = \frac{a(1 - e^2)}{1 + e\sin\theta}.$$
So, $a(1 - e^2) = 2$ and $a = 32/15$. This may also be found from the above figure. We may use the formulas from standard ellipse formula in which a directrix has length a^2/c and center-focus distance c. We are given
$$\frac{a^2}{c} - c = 8 \quad \text{or} \quad \frac{a^2}{ae} - ae = 8.$$
This gives us $a(1 - e^2) = 8e = 2$.
P.S.

The ellipse centered at (x_0, y_0) whose semi major axis has length a and center-focus distance c where $c = \sqrt{a^2 - b^2}, (a > b)$ has equation
$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1.$$
This ellipse has vertices $(x_0 \pm a, y_0)$ and co-vertices $(x_0, y_0 \pm b)$, directrices $x = x_0 \pm \frac{a^2}{c}$ and $e = c/a$.