

You will not get any points if your answer is wrong, that is no points to your explanations if your answer is wrong. And of course no points to a correct answer if your explanation or proof is not correct or clear.

YOU must write GOOD Mathematics

1. Find the equation of the plane through the points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$.

SOLUTION:

Let A, B, C be the respective points. Then

$$\vec{AB} = (0-1)\underline{i} + (1-0)\underline{j} + (0-0)\underline{k} \text{ and}$$

$$\vec{AC} = (0-1)\underline{i} + (0-0)\underline{j} + (1-0)\underline{k}$$

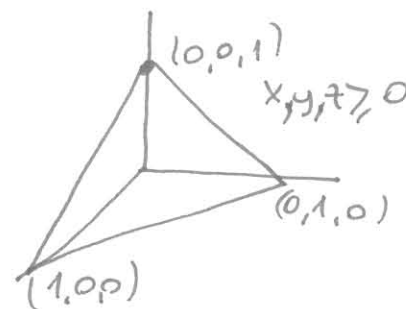
The normal vector for the plane is

$$\vec{n} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \underline{i} - (-1)\underline{j} + \underline{k} = \underline{i} + \underline{j} + \underline{k}$$

Let $P(x,y,z)$ be an arbitrary point on the plane. Then

we have $\vec{PA} \cdot \vec{n} = 0$, i.e. $(x-1) \cdot 1 + (y-0) \cdot 1 + (z-0) \cdot 1 = 0$

$$\boxed{x+y+z=1}$$



2. Find the distance between the plane above and the point $(2,2,4)$.

SOLUTION:

Let S be the point $(2,2,4)$ and consider the vector

$$\vec{AS} = (2-1)\underline{i} + (2-0)\underline{j} + (4-0)\underline{k} = \underline{i} + 2\underline{j} + 4\underline{k}$$

project this vector on \vec{n} and then find its length.



$$d = \frac{|\vec{AS} \cdot \vec{n}|}{|\vec{n}|} = \frac{|1 \cdot 1 + 2 \cdot 1 + 4 \cdot 1|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{7}{\sqrt{3}}$$