
You will not get any points if your answer is wrong, that is no points to your explanations if your answer is wrong. And of course no points to a correct answer if your explanation or proof is not correct or clear.

YOU must write GOOD Mathematics

1. Label the following statements as true or false.

- (a) There exists a linear operator T with no T -invariant subspace.
- (b) Let T be a linear operator on an n -dimensional vector space. Then there exist a polynomial $g(t)$ of degree n such that $g(T) = T_0$.
- (c) Any polynomial of degree n with leading coefficient $(-1)^n$ is the characteristic polynomial of some linear operator.

2. Let A be the matrix

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

and $g(t) = t^2 - 4t - 5$. Verify that $g(A) = \mathbf{0}$.

Solution: