Name-Surname:
Student No: **Class:** **Grade:**

MAT435 **Computer Aided Geometric Design**

Exam 1

09.12.2003

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1. Given $n+1$ points $\mathsf{b}_0, \mathsf{b}_1, \ldots, \mathsf{b}_n$ in the Euclidean space $\mathbb{E}^2$ or $\mathbb{E}^3$, set $\mathsf{b}_i^0 = \mathsf{b}_i, i = 0, 1, \ldots, n$, the *de Casteljau algorithm* is

$$\mathsf{b}_i^r(t) = (1-t)\mathsf{b}_i^{r-1}(t) + t\mathsf{b}_{i+1}^{r-1}(t) \quad \begin{cases} r = 1, 2, \ldots, n \\ i = 0, 1, \ldots, n-r. \end{cases}$$

Prove by induction on r that the intermediate de Casteljau points b_i^r can be represented by

$$\mathsf{b}_i^r(t) = \sum_{j=0}^{r} \mathsf{b}_{i+j} B_j^r(t), \quad r \in \{0, 1, \ldots, n\},\ i \in \{0, 1, \ldots, n-r\}.$$

2. Find the transformation matrix $\mathbf{T}$ between the monomial basis $\Psi = \{1, t, \ldots, t^n\}$ and Bernstein-Be'zier basis $\Phi = \{B_0^n(t), B_1^n(t), \ldots, B_n^n(t)\}$ of $\mathcal{P}_n$ such that $\Psi = \mathbf{T}\Phi$, where

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}, \quad i = 0, 1, \ldots, n, \quad t \in [0, 1].$$

3. Let $\mathsf{b}_0 = (-1, 0), \mathsf{b}_1 = (-3, 3), \mathsf{b}_2 = (3, 3), \mathsf{b}_3 = (1, 0)$ be the control points and $\mathsf{P}(t)$ be Bézier curve. Degree elevate the points and hence find a new set of control points without disturbing the curve. Draw both of the control polygons.

4. Find explicitly the blossom representation of a cubic Bézier curve.