Name-Surname:
Student No: **Class:** **Grade:**

MAT435 **Computer Aided Geometric Design**

Answers for Quiz 1

07.10.2003

- **YOU must write GOOD Mathematics, clearly explaining each step of your proof. Otherwise, no objection!**

1. Find the *affine* mapping which transforms the *line* between the points (-1,-1) and (-1,1) to the *line* between the points (0,0) and (1,1).
SOLUTION: An affine transformation $\Phi\mathbf{x}$ in $\mathbb{R}^n$ is of the form $A\mathbf{x}+\mathbf{v}$, where A is an $n\times n$ matrix and $\mathbf{v}$ is the translation vector. In the question above, $\mathbf{v}=(1,1)^T$ since the line moves 1 unit up and 1 unit right. The parametric equations of the lines are as follows:

$$\begin{aligned}\mathbf{x} &= (1-t)(-1,-1)^T + t(-1,1)^T = (-1,-1+2t)^T\\ \Phi\mathbf{x} &= (1-t)(0,0)^T + t(1,1)^T = (t,t)^T\end{aligned}$$

and is true for any $0\leqslant t\leqslant 1$. Thus we have a system of equations

$$\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}-1\\ -1+2t\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}t\\t\end{bmatrix}$$

whose unknowns are the elements of A. We need 4 equations. Setting $t=0$ gives

$$-a_{11}-a_{12}=-1,\quad -a_{21}-a_{22}=-1$$

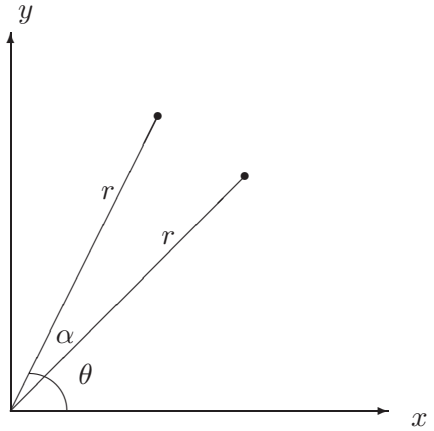
and $t=1$ gives $-a_{11}+a_{12}=0,\quad -a_{21}+a_{22}=0$. Thus, we obtain $a_{11}=a_{12}=a_{21}=a_{22}=\frac{1}{2}$ and hence

$$\Phi\mathbf{x}=\begin{bmatrix}1/2 & 1/2\\ 1/2 & 1/2\end{bmatrix}\mathbf{x}+\begin{bmatrix}1\\1\end{bmatrix}.$$

2. Let $(x,y)\in\mathbb{E}^2$. Derive the *rotation* matrix (an *affine* mapping) which transforms (x,y) to $(\widehat{x},\widehat{y})$ by α degrees counterclockwise. *Hint: Polar coordinates*
SOLUTION:

Let $(x,y)\longleftrightarrow(r,\theta)$ so that $x=r\cos\theta$ and $y=r\sin\theta$. Then we have

$$\begin{aligned}\widehat{x}=r\cos(\theta+\alpha) &= r\cos\theta\cos\alpha - r\sin\theta\sin\alpha\\ &= x\cos\alpha - y\sin\alpha\\ \widehat{y}=r\sin(\theta+\alpha) &= r\sin\theta\cos\alpha + r\cos\theta\sin\alpha\\ &= y\cos\alpha + x\sin\alpha\end{aligned}$$

It follows from above that

$$\begin{bmatrix}\widehat{x}\\ \widehat{y}\end{bmatrix}=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}.$$