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MATH 435 COMPUTER AIDED GEOMETRIC DESIGN **Answers for Quiz 4** 23.12.2003

Name **Student No.**

YOU must write GOOD Mathematics, clearly explaining each step of your proof. Otherwise, no objection will be accepted.

1. Given the function $f(x) = \dfrac{1}{1+x^2}$. Use **Aitken's algorithm** to interpolate the function at 3 equidistant knots of the interval $[-1, 1]$.

SOLUTION The **Aitken's algorithm** which uses successive linear interpolation at parameter values is

$$\mathsf{p}_i^r(t) = \frac{t_{i+r} - t}{t_{i+r} - t_i}\mathsf{p}_i^{r-1}(t) + \frac{t - t_i}{t_{i+r} - t_i}\mathsf{p}_{i+1}^{r-1}(t).$$

Let us take the points $t_0 = -1, t_1 = 0$ and $t_2 = 1$. The corresponding function values are

$$f(t_0) = 1/2, \quad f(t_1) = 1, \quad f(t_2) = 1/2.$$

Thus the algorithm gives,

$$\mathsf{p}_0^1(t) = \frac{t_1 - t}{t_1 - t_0}\mathsf{p}_0^0 + \frac{t - t_0}{t_1 - t_0}\mathsf{p}_1^0 = 1 + \frac{t}{2} \quad \text{and} \quad \mathsf{p}_1^1(t) = \frac{t_2 - t}{t_2 - t_1}\mathsf{p}_1^0 + \frac{t - t_1}{t_2 - t_1}\mathsf{p}_2^0 = 1 - \frac{t}{2}$$

and

$$\mathsf{p}_0^2(t) = \frac{t_2 - t}{t_2 - t_0}\mathsf{p}_0^1(t) + \frac{t - t_0}{t_2 - t_0}p_1^1(t) = 1 - \frac{t^2}{2}.$$

2. Given an nth degree Bézier curve $\mathsf{P}(t)$ where $t \in [0, 1]$. Define the functional form of $\mathsf{P}(x)$ for $x \in [a, b]$.

SOLUTION The functional form a Bézier curve is obtained by taking particular points $\mathsf{b}_i = (i/n, b_i)$ and hence

$$\mathsf{P}(t) = \sum_{i=0}^{n} \mathsf{b}_i B_i^n(t) \quad \text{where} \quad B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}.$$

The affine transformation $t = \frac{x-a}{b-a}$ maps $t \in [0, 1]$ to $x \in [a, b]$. Transforming the point b_i onto $[a, b]$ gives $\mathbf{b}_i = ((b-a)i/n, y_i)$ and hence

$$\mathsf{P}(x) = \sum_{i=0}^{n} \mathbf{b}_i B_i^n(x).$$